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Pond heat and temperature regulation (PHATR): Modeling temperature and energy balances in earthen outdoor aquaculture ponds

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Abstract

A pond heat and temperature regulation (PHATR) model was designed to: (1) predict the temperature for earthen outdoor aquaculture ponds and (2) determine the size of energy transfer mechanisms affecting energy gains or losses for these ponds. The model solves a first order, no-linear differential equation using a 4th order Runge-Kutta numerical method and various input data (weather data, pond characteristics and flow rate data). Output data (predicted pond temperature) was compared to measured pond temperature collected from the warmwater ponds at the Louisiana State University Agricultural Center Aquaculture Research Station, Baton Rouge, Louisiana. The model over-predicted the temperature for unheated ponds by 0.7 °C and for heated ponds by 2.6 °C. Fluctuations in flowrates of warm water used to heat the pond are believed to be responsible for the greater error in predicting heated pond temperatures. On average, the two most important energy vectors for unheated ponds were longwave pond radiation (39%) and longwave sky radiation (31%). At certain times, solar radiation accounted for as much as 49% of all energy transferred to unheated ponds. For heated ponds, on average, important energy transfer mechanisms were longwave pond radiation (25%), longwave sky radiation (19%), warm geothermal-well water (19%) and discharged water (15%). At certain times, solar radiation accounted for as much as 50% and warm well water 60% of all energy transferred to heated ponds.

Keywords: Temperature control; Modeling; Pond energy balance; Aquaculture

1. Introduction

Temperature is a critical water quality parameter in aquaculture. Because fish are ectothermic, temperature affects their growth rate (Davis, 1961; Galtsoff, 1964), spawning cycles (Bye, 1984; Arnold, 1988; Lang et al., 2003; Morrison and Smith, 1986), health (Avault, 1996)

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and survival (Avault and Shell, 1968; Chamberlain et al., 1980). Temperature also strongly influences the concentration of oxygen dissolved in water (Lawson, 1995). For these reasons, it is in the farmer's interest to control the water temperature. For indoor operations, heaters and chillers are commercially available to do this. For outdoor systems, such as earthen ponds, controlling the water temperature is more difficult. The heating of a 400 m^3 catfish pond from 15 °C to 27 °C, as was done by Hall et al. (2002), theoretically requires 20160 MJ (5600 kWh) net. This calculated value does not account for heat losses to the environment (air convection, soil conduction, evaporation, back radiation). In designing temperature control devices for outdoor aquaculture ponds, these losses must be included in calculations.

The purpose of this study was (1) to accurately predict the pond temperature for heated and unheated research-sized aquaculture ponds and (2) to determine the magnitude of energy transfer mechanisms for such ponds by developing an energy balance model. The energy balance model pond heat and temperature regulation (PHATR) is a computer program designed specifically for this purpose.

2. Pond description

Model results were compared to actual temperature data, collected from the outdoor earthen warm water ponds at the Louisiana State University Agricultural Center Aquaculture Research Station (ARS) (geographic coordinates: 30° 21′N, 91°10′W). The 13 warm water ponds were used between January and June to spawn channel catfish (*Ictalurus punctatus*) before and during the natural spawning season (May–June). Channel catfish require the pond temperature to be above 24 °C to spawn (Source). Because these ponds have access to warm geothermal water (36 °C), they can be heated to the desired temperature at any time of year.

The 13 warm water ponds were earthen with the pond bottom soil classified as a Sharkey Dundee clay. Twelve ponds (Fig. 1) were roughly 10 meters by

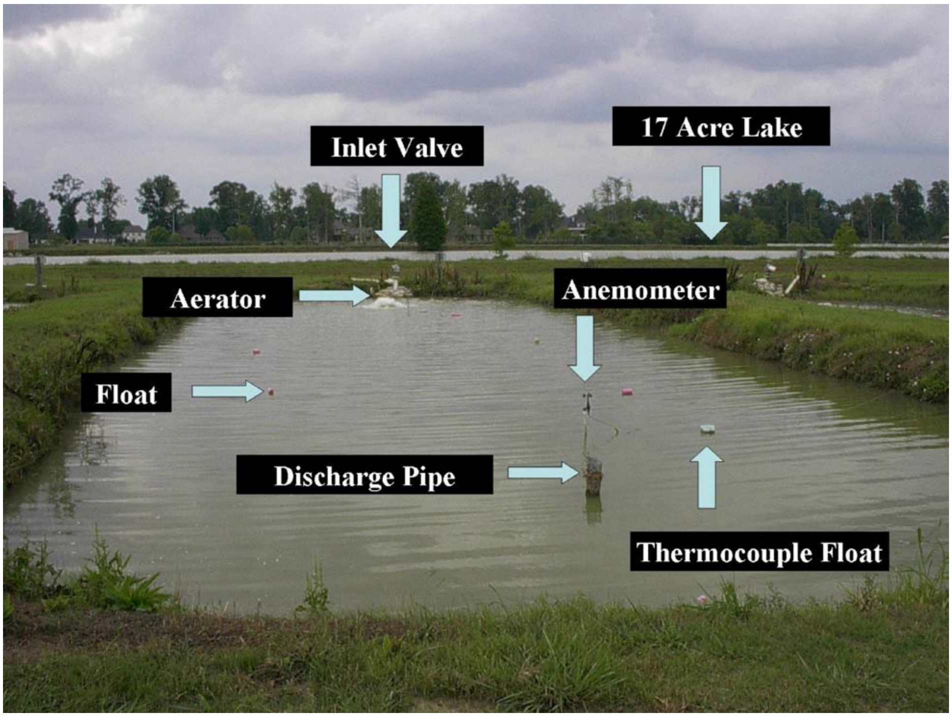

Fig. 1. Pond 6, shown facing North, was one of the 13 warmwater ponds at the ARS. Ponds 1–12 were approximately the same size (average area, 330 m^2, average volume, 400 m^3). Each pond was equipped with an aerator and inlet valves for the addition of cold and warm water. The discharge pipe was at the South end of the pond. Floats marked the location of catfish spawning containers on the pond bottom. An anemometer was installed to measure wind speed. A type-T thermocouple was suspended 10 cm below the water surface from a float to measure pond temperature.

30 meters (average surface area: 330 m^2; average pond volume: 400 m^3) and the 13th pond (Pond 13) was trapezoidal (surface area: 390 m^2; volume: 480 m^3). Each pond had access to warm geothermal water (36 °C) and cold well water (21 °C). Each pond had a discharge pipe which maintained the pond depth at 1.22 m. An aerator (Power House, 0.56 kW) was located in each pond approximately 2 m from the inlets, both of which were at the North end of the pond. The warm water was drawn from a 700-m deep geothermal well. The pump, rated at 30 kW (40 hp), delivered water at a flow rate of 2400 lpm.

The pond temperature was controlled with a solenoid ball valve (models LB308 and E153; Hayward Industrial Products, Elizabeth, N.J.) connected to a data logger (Campbell Scientific CR 23X, Campbell Scientific Inc., North Logan, UT). In response to low temperatures automatically measured with a type-T (copper-constantan) thermocouple, the data logger sent a 5 mV signal to open a solenoid valve, allowing warm water to flow into the pond. Once the desired temperature was attained, the valve was closed until the pond cooled to the minimum set temperature. Below this point, the valve opened again. The temperature in four of the 12 ponds was controlled at a time. The other eight ponds were not heated. The control system has been described in greater detail (Hall et al., 2002). To induce channel catfish spawning, the pond temperature was maintained between 26 °C and 27 °C.

Data loggers (Campbell Scientific 21X, Campbell Scientific Inc., North Logan, UT) measured the pond temperature using a type-T thermocouple suspended 10 cm from a float. The float was located at the pond surface within 2 m of the pond discharge pipe. The flow rate for each combination of open and closed valves was measured by measuring the time it took to fill a 120 l bucket. By knowing which combination of valves were open and closed, the flow rate at any time to any pond was known. The Campbell CR23X recorded when valves opened and closed.

3. Description of PHATR equations

Mathematically, PHATR solved Eq. (1), the differential equation which describes changes in internal energy for fully mixed outdoor earthen aquaculture ponds.

$$\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_{\mathrm{pond}} = q_{\mathrm{solar}} - q_{\mathrm{back}} + q_{\mathrm{sky}} - q_{\mathrm{evap}} \pm q_{\mathrm{conv}} \pm q_{\mathrm{soil}} - q_{\mathrm{seep}} + q_{\mathrm{rain}} + q_{\mathrm{well}} - q_{\mathrm{out}} \pm q_{\mathrm{other}} \quad (1)$$

where E is the internal energy (J) at any given time (t) in the pond; q_{solar} is the rate of energy gained by the pond by solar radiation (W); q_{back} is the rate of heat loss due to longwave back radiation (W); q_{sky} is the rate of energy gained by longwave sky radiation (W); q_{evap} is the rate of energy lost through the evaporation of water (W); q_{conv} is the rate of heat exchanged with the air by convection (W); q_{soil} is the rate of heat exchanged with the soil (W); q_{seep} is the rate of bulk energy lost through seepage (W); q_{rain} is the rate of bulk energy gained due to rainfall (W); q_{well} is the rate of bulk energy gained from the warm water well (W); q_{out} is the rate of bulk energy lost to the overflow of water (W); q_{other} is the rate of energy transfer from or to other sources (W).

Fig. 2 is a conceptual representation of Eq. (1), the energy balance.

The amount of internal energy inside the control volume (the pond) is:

$$E_{\mathrm{pond}} = \rho \forall c_p T \quad (2)$$

where ρ is the density of the water (kg/m^3); $\forall$ is the volume of water in the pond (m^3); c_p is the specific heat of water (kJ/kg °C) and T is the average temperature (°C).

Because of a continuously running aerator, the pond was assumed ideally mixed and the temperature was assumed the same throughout the pond. (This is supported by experimental data; Lamoureux, 2003.) The pond volume was assumed to be constant, despite leaks and evaporation. When water was added to the ponds, a standpipe ensured that the water depth never rose above 1.2 m. Water properties such as density and specific heat were also assumed to be constant with temperature. At 0 °C, the density of water is 999.8 kg/m^3 and at 43.3 °C, the density is 990.6 kg/m^3 (less than 1% change). At 0 °C, the specific heat is 4225 J/kg°C and at 43.3 °C, the specific heat is 4174 J/kg°C (a relative change of 1.2%) (Holman, 1997).

Solar radiation, q_{solar}, was measured directly from field observations. The recorded data were used

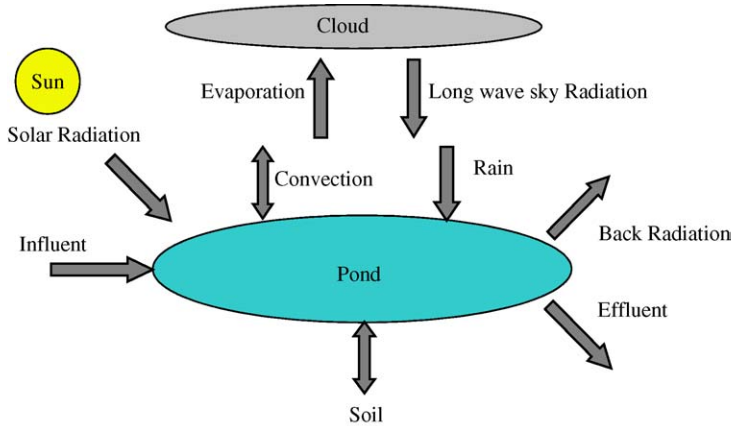

Fig. 2. This energy balance represents the conceptual version of PHATR. Each arrow represents an energy vector (a "q" term in Eq. (1)). Two-headed arrows represent transport phenomena which either move energy into the pond (when the surroundings are warmer than the pond) or out of the pond (when the surroundings are cooler than the pond). By taking the sum of all vectors, the rate of change for both the pond's internal energy and the pond temperature were determined.

directly by PHATR as input data during model runs. Unlike pure water (a good absorber of infrared radiation but a poor absorber of visible light), pond water was assumed to absorb all solar radiation. The extinction coefficient of pond water was measured as 0.013 mm^{-1} (Lamoureux, 2003).

Noting that the emissivity of water is 0.96 (Siegel and Howell, 1981), the rate of heat loss due to longwave pond radiation is:

$$q_{\text{back}} = 0.96 A_{\text{pond}} \sigma (T_{\text{air}} + 273)^4 \tag{3}$$

where q_{back} is the backradiation of the pond (W); 0.96 is the emissivity of water (Siegel and Howell, 1981); Φ is t he Stefan-Boltzmann constant (5.67×10^{-8} W/m^2/K^4); A_{pond} is the pond area (m^2); T_{pond} is the temperature of the pond (K).

When calculating the atmospheric longwave radiation during model runs, the sky was always assumed to be cloudless. For Baton Rouge, Louisiana (annual average rainfall: 1600 mm; Anonymous, 2003), this may not have been a valid assumption. However, a cloudless sky emits less longwave radiation than an overcast sky containing lots of moisture (Bliss, 1961). Therefore, assuming a cloudless sky ensured that the model did not under predict the amount of energy required to heat a pond. For a cloudless sky, the apparent sky emissivity can be estimated with the following equation (compiled from Bliss, 1961):

$$\varepsilon_{\text{sky}} = \frac{1}{c_1 - c_2 T_{\text{dew}} + c_3 T_{\text{dew}}^2} \tag{4}$$

where $c_1 = 1.2488219$; $c_2 = -0.0060896701$; $c_3 = 4.8502935 \times 10^{-5}$; T_{dew} is the dew temperature (°C).

The dew temperature was calculated with the following equation (compiled from Anonymous, 1992):

$$T_{\text{dew}} = \frac{1}{1/T_{\text{air}} + 273 - (1.846 \times 10^{-4}) \ln(\text{rh})} - 273 \tag{5}$$

where T_{air} is the air temperature (°C); rh is the relative humidity (decimal)

Using the apparent emissivity, the longwave sky radiation was calculated as (Bliss, 1961):

$$q_{\text{sky}} = \varepsilon_{\text{sky}} A_{\text{pond}} \sigma (T_{\text{air}} + 273)^4 \tag{6}$$

where T_{air} is the air temperature (K).

Evaporation heat losses (q_{evap}) were calculated with the following set of equations (Anonymous, 1992):

$$q_{\text{evap}} = \dot{m}_{\text{evap}} h_{\text{fg}} = E_0 \rho_{\text{water}} h_{\text{fg}}, \tag{7}$$

$$h_{\text{fg}} = 2,502,535.259 - 212.56384T \tag{8}$$

where $\dot{m}_{\text{evap}}$ is the rate of evaporation (kg/s); h_{fg} is the latent heat of vaporization (J/kg); E_0 is the volumetric rate of evaporation (m^3/s); ρ_{water} is the density of water (kg/m^3); T is the water temperature (°C).

The Lake Hefner Equation (Anonymous, 1952) was used to predict the volumetric evaporation rate:

$$E_0 = (0.068 + 0.13u_{4m})(p_{sat\text{-}wv} - p_{vp}) \times (1.344 \times 10^{-7}) \quad (9)$$

where E_0 is the evaporation rate (m/s); u_{4m} is the wind speed recorded at 4 meters (m/s); $p_{sat\text{-}ws}$: is the saturated vapor pressure (Pa); p_{vp} is the air vapor pressure (Pa); In order to determine the wind speed at any height, the following equation was used:

$$\frac{u_y}{u_x} = \frac{\ln y}{\ln x} \quad (10)$$

where u is the wind speed at either height x or y (feet); It was assumed there was no evaporation when the relative humidity of the air was at or above 100%.

Heat transferred through convection was calculated using Newton's Law of cooling:

$$q_{conv} = hA(T_{surface} - T_{fluid}) \quad (11)$$

where q_{conv} is the heat transferred by convection (W); h is the heat transfer coefficient (W/m^2 K); A is the area of heat transfer (m^2); $T_{surface}$ is the temperature of the surface (°C or K); T_{fluid} is the temperature of the cooling (or heating) fluid (°C or K).

The formula used to calculate the heat transfer coefficient, as a function of wind speed, was (McAdams, 1942; Watmuff et al., 1977):

$$h = 2.8 + 3.0V \quad (12)$$

where V is the wind velocity (m/s).

For soils, conduction was experimentally verified to be the predominant mode of heat transfer (Kimball et al., 1976). Consequently, the rate at which heat is exchanged with the soil (q_{soil}) was described by Fourier's Law of heat conduction:

$$q_{soil} = -k_{soil}A\left.\frac{\partial T}{\partial z}\right|_{z=0} \quad (13)$$

where k is the soil thermal conductivity (W/m K); A is the pond floor area (m^2); T is the temperature of the soil (°C); z is the soil depth (m); $(\subset T/\subset z)|_{z=0}$ is the temperature gradient at the pond floor.

The temperature gradient, in turn, was determined from solutions to the one-dimensional Heat Diffusion Equation:

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial z^2} \quad (14)$$

where t is time (s); $\forall$ is the soil's thermal diffusivity (m^2/s).

Because it was assumed to be constant, the thermal diffusivity was calculated from other soil parameters:

$$\alpha = \frac{k_{soil}}{\rho_{soil}c_{p\text{-}soil}} = \frac{k_{soil}}{C_v} \quad (15)$$

where k_{soil} is the soil's thermal conductivity (W/m/K); ρ_{soil} is the soil's bulk density (kg/m^3); $c_{p\text{-}soil}$ is the soil's specific heat (J/kg/K); C_v is the soil's volumetric specific heat (J/m^3/K).

Table 1 lists values from the literature describing the thermal parameters of heavy soils. Because the pond bottom was compacted Sharkey-Dundee clay (compacted to prevent leaks), and because the soil was fully saturated with water, these soil properties were assumed to be constant, regardless of position or depth.

To solve the Heat Diffusion Equation, one initial and two boundary conditions are required. For the initial condition, it was assumed that the temperature throughout the entire soil was initially the same at all depths:

$$T_{soil}(z, t) = T_{initial} \quad (16)$$

Table 1
These thermal soil properties, found in the literature, are of relevance to this study

Soil	k_{soil} (W/m K)	C_v (MJ/m^3K)	Source	Note
Clay minerals	2.92	2	de Vries (1966)	Property evaluated at 10 °C
Organic matter	0.25	2.51	de Vries (1966)	Property evaluated at 10 °C
Silty clay loam	1.45–2.07	1.6–2.05	Sikora et al. (1990)	Severely compacted soil
Saturated clay	1.6	2.9	Kimball (1983)	Soil has a porosity of 0.4.

For unheated ponds, the two boundary conditions were (Van Wijk and de Vries, 1966):

$$\lim_{z\to\infty} T_{\text{soil}}(z,t) = T_{z=\infty},$$
$$T(z=0,t) = T_{\text{avg}} + T_{\text{amp-day}}\sin(\omega_{\text{day}}t) + T_{\text{amp-yr}}\sin(\omega_{\text{yr}}t) \quad (17)$$

$$T(z=0,t) = T_{\text{avg}} + T_{\text{amp-day}}\sin(\omega_{\text{day}}t) + T_{\text{amp-yr}}\sin(\omega_{\text{yr}}t) \quad (18)$$

where T_{avg} is the average soil surface temperature for the period $1/T$ (°C); T_{amp} is half the total variation of the average temperature(°C); ω is the frequency of the period being considered (s^{-1}).

Solving the Heat Diffusion Equation (Eq. 14) yielded the soil temperature profile (solution presented by Van Wijk and de Vries, 1966). Substituting the derivative of the soil temperature profile at the surface $(\subset T/\subset z)|_{z=0}$ into Fourier's Law of Heat Conduction (Eq. 13) yielded the heat exchanged between the soil and the pond (q_{soil}):

$$q_{\text{soil}} = -k_{\text{soil}}A_{\text{pond}}\left\{\begin{array}{l} -T_{\text{amp}\,z=0\,\text{yr}}\dfrac{1}{D_{\text{yr}}}\left[\sin\left(\omega_{\text{yr}}t+\phi_{z=0\,\text{yr}}\right)+\cos\left(\omega_{\text{yr}}t+\phi_{z=0\,\text{yr}}\right)\right] \\ -T_{\text{amp}\,z=0\,\text{day}}\dfrac{1}{D_{\text{day}}}\left[\sin\left(\omega_{\text{day}}t+\phi_{z=0\,\text{day}}\right)+\cos\left(\omega_{\text{day}}t+\phi_{z=0\,\text{day}}\right)\right] \end{array}\right\} \quad (19)$$

where ϕ is the phase constant for temperature variations at $z = 0$ m; D is the dampening depth (m).

The dampening depth (D) is:

$$D = \sqrt{\frac{2k}{C_v\omega}} \quad (20)$$

where k is the thermal conductivity (W/m K); C_v is the soil's volumetric specific heat (MJ/m^3 K) and $1/T$ is the period of the variation being considered (s).

For the case, where the water temperature remained constant (controlled), the second boundary condition was:

$$T(z=0,t) \approx T_{\text{pond}} \quad (21)$$

The solution to the Heat Diffusion Equation (the soil temperature profile), with Eqs. (17) and (21) as boundary conditions, was presented by Holman (1997). Substituting the derivative of the soil temperature profile at the surface $(\subset T/\subset z)|_{z=0}$ into Fourier's Law of Heat Conduction (Eq. (13)) yielded the heat exchanged between the soil and the pond (q_{soil}):

$$q_{\text{soil}} = kA\frac{(T_{\text{pond}} - T(z,t=0))}{\sqrt{\pi\alpha t}} \quad (22)$$

For all bulk movements of water, there is also an associated movement of internal energy. The rate of bulk energy transferred across the system boundary was calculated with the following equation:

$$q = \dot{m}c_pT \quad (23)$$

where $\dot{m}$ is the mass flow rate of water into (or out of) the system (kg/s); c_p is the specific heat of water and (J/kg°C); T is the temperature of the water (°C).

Losses due to seepage were ignored because of the practical difficulties involved in measuring its mass flow rate.

Other sources of energy such as the respiration of decomposing bacteria on the pond bottom or energy from a continuously operating aerator were considered negligible for this study.

4. Description of PHATR—programming logic

PHATR was developed in FORTRAN, using the Essential Layhey FORTRAN 90 (ELF 90) compiler (manufactured by Lahey, www.lahey.com). PHATR followed these steps to (1) perform an energy balance and (2) determine the pond temperature:

4.1. Data input

PHATR retrieved information about the weather, environmental constants and the water flow rate from ASCII files. The weather data (parameters of interest included air temperature, solar radiation, wind speed and relative humidity), available at the Louisiana Agriclimatic Information Web Site (www.lsuagcenter.com/weather), were collected at the Ben Hur Weather Station, less than 1 km from the ARS warm water ponds.

4.2. Determination of vectors

After reading the input data (weather conditions, flow rates, environmental constants), PHATR determined the magnitude of each energy vector. (The vector terms were "q"s in Eq. (1) and arrows in Fig. 2.)

4.3. Solving Eq. (1)—performing the energy balance

By substituting the vectors into Eq. (1), the rate of change of the pond temperature was:

$$\left(\frac{dT}{dt}\right)_{\text{pond}} = \frac{\sum q}{\rho \forall_{\text{pond}} c_{\text{p}}} = f(t, T) \tag{24}$$

where T is the temperature (°C); t is time (s); q is an energy vector in Eq. (1) (W); ρ is the density of water (kg/m^3); $\forall_{\text{pond}}$ is the pond volume (m^3); c_p is the specific heat of water (J/kg°C)

To solve this differential equation, the 4th order Runge-Kutta numerical technique (Cheney and Kincaid, 1985) was used. Calculations were repeated for every hour (time step = 1 h).

4.4. Data output

The time, the vectors and the pond temperature were all recorded in an output ASCII file. Steps 1 through four were repeated for a desired time period. Once the model run was completed, the output was compared to the measured data in a spreadsheet program.

5. Description of model runs

Model runs were performed for ponds 5, 6, 7 and 8 (from 04/03/02 to 04/07/02), pond 13 (from 11/09/02 to 12/16/02 and from 12/25/02 to 01/24/03) and ponds 3, 9 and 12 (from 02/13/03 to 03/23/03). For the presented model runs, ponds 3, 5, 6, 7, 8 and 13 were unheated while ponds 9 and 12 were heated.

To compare the modeled and measured data, the following three statistical parameters were used:

1. The average bias: bias is a measurement of how accurate the model was at estimating the actual pond temperature (i.e. the modeled temperature minus the measured temperature). The average bias is the average of all the biases at every time step, for n time steps:

$$\text{Average bias}(\mu) = \frac{\sum(T_{\text{model}} - T_{\text{measured}})}{n}$$

2. The standard deviation of the average bias: the standard deviation measured variations in the bias. It is an indicator of the model's consistency.

$$\text{S.D.} = \sqrt{\frac{\sum(T_{\text{model}} - T_{\text{measured}} - \mu)^2}{n-1}}$$

3. The correlation coefficient: the linear correlation coefficient, r, between the measured and modeled temperatures is another indicator of the model's ability to predict changes in water temperature. When $|r| = 1$, the model predicted changes in pond temperature perfectly. When $|r| = 0$, the model failed completely to predict pond temperature changes.

6. Selected results

Results for the comparison between measured and modeled pond temperatures for all 9 model runs have been tabulated in Tables 2 and 3. The measured and modeled pond temperatures for the 1st Pond 13 model run are shown in Figs. 3 and 4. The average contribution for each energy vector (for unheated and heated ponds) is shown in Figs. 5 and 6.

Table 2
Measured pond data was compared to modeled data, generating the statistical parameters evaluating PHATR's ability to predict the temperature in unheated ponds

Pond number	Average bias (°C)	Standard deviation (°C)	Correlation coefficient (r)
5	0.2	0.9	0.96
6	0.5	0.9	0.97
7	0.7	0.6	0.99
8	0.6	0.7	0.97
13 (1st run)	1.6	1.1	0.95
13 (2nd run)	1.0	1.2	0.85
3	0.2	2.0	0.94
Average	0.7	1.1	0.95
Weighted average	0.9	1.3	0.92

Because different model runs examined the effects of energy transfer over different lengths of time, weighted averages for each vector were calculated, giving more importance to runs with longer time spans.

Table 3
Measured pond data was compared to modeled data, generating these statistical parameters describing PHATR's accuracy when predicting the temperature in heated ponds

Pond number	Average bias (°C)	Standard deviation (°C)	Correlation coefficient (r)
9	2.2	1.1	0.83
12	2.6	1.6	0.92
Average	2.4	1.25	0.875

6.1. Predicting temperature in unheated ponds

For unheated ponds, the model had a tendency to over-estimate the temperature by 0.7 °C (standard deviation, 1.1 °C; Table 2). This tendency to over-predict may be explained in three ways:

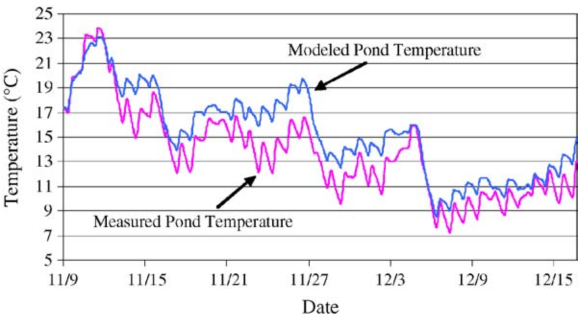

Fig. 3. The modeled and measured pond temperatures for unheated Pond 13 (11/13/02 to 12/16/02) were plotted against time. For this run, the model had a tendency to over-predict the pond temperature (average bias of 1.6 °C) but the shape of the two curves was similar (correlation coefficient, $r = 0.95$).

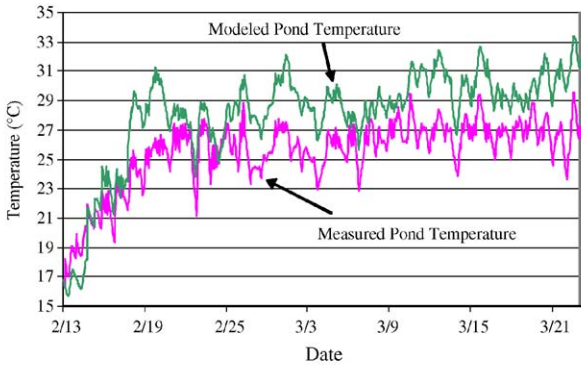

Fig. 4. The modeled and measured pond temperatures for heated Pond 12 (02/13/03 to 03/23/03) were plotted against time. For this run, the model had a tendency to over-predict the pond temperature (average bias of 2.6 °C) but the shape of the two curves was similar (correlation coefficient $r = 0.92$).

(1) PHATR was sometimes over-estimating energy vectors entering the pond. Such energy vectors included solar radiation and longwave sky radiation. By over-estimating the amount of energy entering the pond, the pond's internal energy and temperature were also over-estimated.

The radiation measurements at the Ben Hur Weather Station were compared to weather measurements from another Baton Rouge Weather Station (Burden; geographic coordinates: 30°24′N, 91°08′W). Radiation readings at the

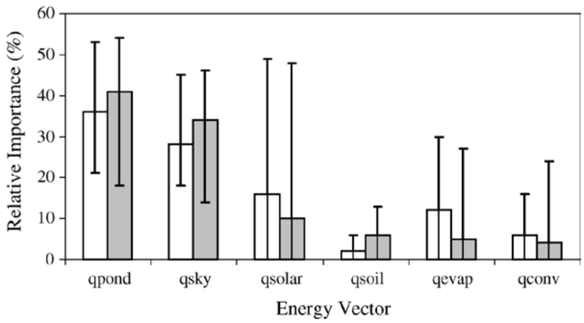

Fig. 5. The relative importance for each energy vector for all unheated pond model runs was compiled into two main categories: spring (white bars) and fall-winter (grey bars). The bars represent the average relative importance while the line extensions represent the range. For instance, in the spring, the average relative importance of solar radiation was 16%; the maximum relative importance was 49%. The minimum values for solar radiation (q_{solar}), soil conduction (q_{soil}), evaporation (q_{evap}) and air convection (q_{conv})were 0%. The other energy transfer mechanisms were longwave pond radiation (q_{pond}) and longwave sky radiation (q_{sky}).

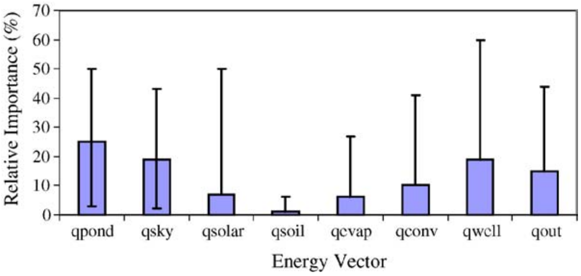

Fig. 6. The relative importance for each energy vector for all heated pond model runs was compiled. The bars represent the average relative importance while the line extensions represent the range. For instance, the average relative importance of solar radiation was 7%; the maximum relative importance was 50%. The minimum values for solar radiation (q_{solar}), soil conduction (q_{soil}), evaporation (q_{evap}), air convection (q_{conv}), warmwater (q_{well}) and discharged water (q_{out}) were 0%. The other energy transfer mechanisms were longwave pond radiation (q_{pond}) and longwave sky radiation (q_{sky}).

Burden Station were lower than those at the Ben Hur Station. For instance, for the month of January, 2003, Ben Hur Station radiation measurements were on average 20W greater than Burden Station radiation measurements. Running PHATR for the second pond 13 model run (12/26/02 to 01/24/03) using Burden Station weather data yielded an average bias of 0.1 °C (standard deviation, 1.1 °C) as opposed to an average bias of 1.0 °C obtained for Ben Hur weather data. These results suggest that input data had an effect on how well PHATR predicted pond temperatures.

Using Eqs. (4–6) to estimate longwave sky radiation assumed that the sky was cloudless and the atmosphere could be considered as stratified layers of different gas mixtures. If this was not the case (because of turbulence from a weather front), then the underlying assumptions used by Bliss (1961) to develop Eq. (4) (the equation used to determine the sky's emissivity) were not valid. However, the presence of clouds would increase the amount of longwave sky radiation captured by the pond. This would cause the model to under-predict the internal energy of the pond and consequently, longwave sky radiation probably not causing the model to over-predict the pond temperature. A pyrgeometer would have to be used to directly measure longwave sky radiation.

(2) PHATR was under-estimating energy losses, perhaps because the evaporation rate was being under-estimated. Evaporation was important to the energy balance. Evaporation, on average, accounted for 12% of all energy in transit for the spring model runs and 5% of all energy in transit in the fall. However, the importance of evaporation was as high as 30% (Fig. 5). Eq. (9), the Lake Hefner Equation, was designed to predict the daily evaporation over a lake using the daily average wind speed (actually, the units for wind speed used in the original reference by Anonymous (1992), were miles per day) and not the instantaneous wind speed recorded every hour as does PHATR. Therefore, to determine if the Lake Hefner Equation was valid, additional model runs were made using other empirical equations from the literature which predict evaporation rates (Penman, 1948; Piedrahita, 1991). Results from these model runs are detailed in Table 4. The average bias, when using the Lake Hefner Equation, was always the median average bias. Although the Lake Hefner Equation is not meant to be precise, it does offer results comparable to other empirical equations and should not be seen as the cause for PHATR's tendency to over-predict pond temperature.

For natural systems, the importance of air convection and soil conduction was between 2% and 6% (see Fig. 5). Because the temperature gradient between the pond and its environment, the driving force behind both heat transfer mechanisms, was small, these vectors were not as important as radiation energy transfer mechan-

Table 4
The use of different empirical equations to predict the evaporation rate affected PHATR's accuracy

Pond number	Treatment	Equation used	Statistics		
			Bias (°C)	S.D. (°C)	r
3	Unheated	Penman (1948)	−0.7	1.8	0.96
3	Unheated	Piedrahita (1991)	0.5	2.1	0.94
3	Unheated	Lake Hefner (1952)	0.3	2.0	0.94
12	Heated	Penman (1948)	1.3	0.8	0.96
12	Heated	Piedrahita (1991)	3.2	1.1	0.95
12	Heated	Lake Hefner (1952)	2.9	1.0	0.95

Statistics for accuracy were: average bias (bias), the standard deviation of the average bias (S.D.) and the correlation coefficient (r). For heated and unheated ponds, use of the Lake Hefner Equation yielded an average bias between the values for runs using Penman's (1948) and Piedrahita's (1948) Equations.

isms (for instance, the average importance for longwave sky radiation ranged from 28% to 35%). Therefore, even if both these vectors were not properly estimated, the weight of their errors was small and should not be used to explain PHATR's tendency to over-predict pond temperature.

(3) PHATR might not have been taking into account other energy transfer mechanisms. Such mechanisms could have been scattered solar radiation which was reflected out of the pond. Light, usually green light, was poorly absorbed by phytoplankton (Hall and Rao, 2001) and water (Siegel and Howell, 1981). Green light (average wavelength at 550 nm, Smith and Cooper, 1957) represented 12% of the total solar radiation or 6% of all energy fluxes into and out of the pond when the importance of solar radiation was at its peak (49% pond 6 and 7), i.e. 6% potential error. Assuming that the pond was absorbing all solar radiation, PHATR would over-predict the pond temperature. Running PHATR for Pond 13 between 11/13/02 and 12/16/02 with modified solar radiation data (88% of the total measured solar radiation data) yielded better results (average bias, 0.9 °C; standard deviation, 1.0 °C).

In addition to the differences in bias, there were other differences between the modeled and measured results. For instance, the diurnal fluctuations in the measured data were greater than those in the modeled data. The thermal mass of Pond 13 may have been over-estimated (there may have been less water in the pond than was assumed). Mathematically, this makes sense. Assuming the pond temperature was a sinusoidal function of time ($f(t)$), the energy in the pond at any given time was ($E = 0$ when $T = 0$ °C)

$$E = \rho \forall c_p T = k f(t) \tag{25}$$

where E is the pond energy (J); ρ is the density of water (kg/m^3); $\forall$ is the pond volume (m^3); c_p: is the specific heat of water (J/kg°C); T is the pond temperature (°C); $f(t)$ is a sinusoidal function of time; k is a constant.

Isolating T yielded:

$$T = \frac{k}{\rho c_p \forall} f(t) \tag{26}$$

where $(k/'c_p \forall)$ is the amplitude of $f(t)$.

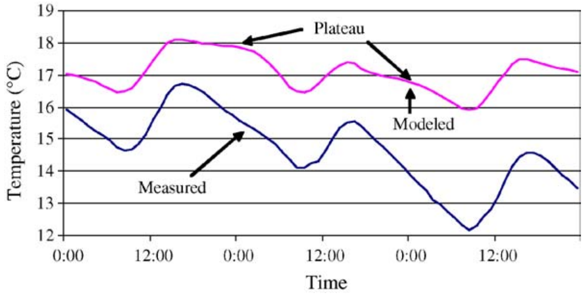

Fig. 7. In addition to over-predicting the actual pond temperature, the modeled curve had a smaller amplitude and several plateaus. The smaller amplitude could be caused by under-estimating the pond volume. The plateaus were caused by not properly modeling soil conduction. These temperatures were modeled and measured for Pond 13 between 11/21/02 and 11/23/02.

If the pond volume was over-estimated, the amplitude of the modeled temperature function ($f(t)$) would be smaller, as was the case shown in Fig. 7.

The shape of the measured and modeled curves was also different. A small plateau, not present in the measured curve, was present in the modeled curve each time the temperature decreased (Fig. 7 and Fig. 8). Closer examination revealed that these changes in slope occurred each night at the same time. This change in slope was caused by not properly modeling soil conduction. The daily phase angle (Eq. 19) which shifted the conduction curve in Fig. 8 caused the pond temperature to decrease just as the heat transfer due to soil conduction reached its peak. Without taking into account the effects of soil heat transfer, the average bias for Pond 13 between 11/09/

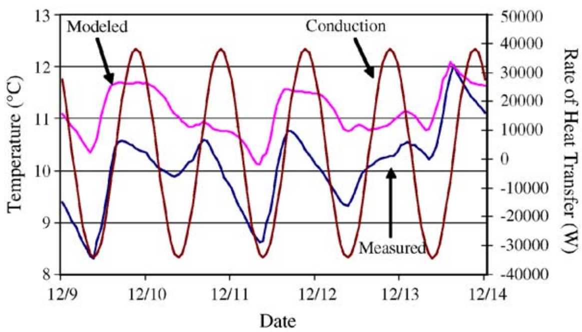

Fig. 8. The soil heat conduction curve was modeled as a sinusoid. As conduction began to decrease, the rate of decrease for the model curve increased. There were no plateaus in the measured pond curve, which indicated that the soil heat conduction curve was out of phase.

Table 5
The use of different empirical equations to predict the convection rate affected PHATR's accuracy

Pond	Treatment	Equation used	Statistics		
			Bias (°C)	S.D. (°C)	*r*
3	Unheated	Watmuff et al. (1977)	0.3	2.0	0.94
3	Unheated	Nusselt correlations	0.4	2.0	0.93
12	Heated	Watmuff et al. (1977)	2.9	1.0	0.95
12	Heated	Nusselt correlations	5.0	1.2	0.95

Statistics for accuracy were average bias (bias), the standard deviation of the average bias (S.D.) and the correlation coefficient (r). For heated ponds, Watmuff et al.'s Equation (1977) was more accurate than the Nusselt number correlations.

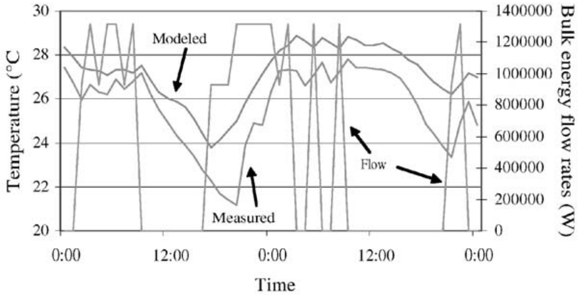

Fig. 9. This data represents the pond temperature and warmwater flow for Pond 12 on 02/22/03 and 02/23/03. During the first afternoon, the measured pond temperature decreased faster than the modeled pond temperatures, revealing that evaporation might have been under-estimated. At 17:00 on the first afternoon, warmwater supposedly flowed into the pond. However, the pond temperature only began to rise at 20:00, when a valve on another pond closed. This meant that water only began to flow into Pond 12 at 20:00, not at 17:00. As can be seen, PHATR's accuracy was dependent on both the quality of the collected data and the validity of its equations.

02 and 12/16/02 was 1.4 °C (standard deviation, 1.0 °C; r, 0.96). Therefore, using a phase angle of $\pi/2$ was not proper. Because there was no change in slope for the measured curve while the temperature dropped (i.e. there did not seem to be a lag between the soil surface maximum temperature and the pond maximum temperature), the daily phase angle should be based on the time when the daily pond temperature maximum occurred. The maximum pond temperature normally occurred between 14:00 and 17:00. Assuming that the maximum occurs at 16:00, the daily phase angle should be $-5\pi/6$ (Table 5).

6.2. Predicting temperature in heated ponds

The average bias for heated pond model runs was 2.4 °C (standard deviation, 1.3 °C; Table 3), higher than the average bias (0.7 °C) for unheated pond model runs (standard deviation, 1.1 °C). In particular, the average bias for the pond 3 model run, performed for the same time period as the heated pond model runs, was 0.2 °C (standard deviation, 2.0 °C). There are four possible reasons for this:

(1) The importance of surface convection and evaporation was greater for heated ponds because of the greater temperature gradient between the pond and the air. Both sets of equations used by PHATR to predict the size of both surface energy vectors were empirical and might not have been accurate when they dominated other modes of energy transfer. This happened at night or on cloudy days when there was little solar radiation and no bulk movements of energy associated with water flows. For instance, on 02/22/2003, the average importance of evaporation between 11:00 and 16:00 was 26% while the average importance of solar radiation was 16% (a cloudy day). During this period of time, the measured temperature declined at a faster rate than the modeled temperature, causing the bias to increase from 0.3 to 1.5 °C (Fig. 9). It was already demonstrated in Table 4 that the Lake Hefner Equation agreed with other empirical equations and that this empirical equation should not be held responsible for inaccuracies in the model results. Extra model runs were, therefore conducted to compare Eq. (12), Watmuff's Equation (1977), with the Nusselt number correlations (Holman, 1997). The average bias using the Nusselt correlations was 5.0 °C. Using Eq. (12) was, therefore more accurate in this case.

(2) The flow of water into and out of the pond varied from the estimated flows used by PHATR. The method used to measure the flow rate of warm water assumed that for a given combination of open and closed valves along the water line, a fixed flow rate would result. However, this was apparently not the case. For the evening of 22nd February, 2003, the well was turned on at 17:00 and according to the recorded data, all four valves

Table 6
If 5 s were measured to fill a 120 l bucket when it really took 4, PHATR under-estimated the energy flow rate by 25%

Time (s)	Flow rate (m^3/s)	Bulk energy flowrate (W)	Relative error
4	0.026	988693	25%
5	0.021	790954	–
6	0.018	659128	17%

If it really took 6 s to fill the bucket, the model over-predicted the bulk energy flow rate by 17%.

were open. Normally, water would flow to all four controlled ponds, including Pond 12. However, the rate of decline for the measured temperature curve at 17:00 did not change. Rather, the pond only began to warm at 20:00, when one of the valves at another pond closed. The bias, by this time, had increased from 1.5 to 3.8 °C. Apparently, no water flowed into Pond 12, despite the open valve.

(3) The method used to estimate the flowrate had measurement error. The flow was measured by timing the period it took to fill a 120 l bucket with 100 l of water. The measurement has an accuracy of ±1 s. When only one valve was open, filling the bucket took 5 s. This translated into a possible relative error of 17% to 24% when estimating the bulk energy flow rate (Table 6). For a flow rate of 0.018 m^3/s, the rate at which the pond heated (dT/dt) was 1.7 °C/h. For a flow rate of 0.026 m^3/s, the rate at which the pond heated up was 2.5 °C/h. Consequently, errors in flow measurements had an effect on the model's bias.

(4) The model time step of 1 h was likely too large. Unlike other energy transfer mechanisms, the flow of warm water was a discrete-event energy transfer vector (i.e. bulk energy flow rates changed within seconds). Because of this, modeling with large time steps was not appropriate. For example, if a valve were to open 50 min before the hour X:00, and PHATR was using data sampled every hour (X:00), PHATR would have ignored the effects of heating the pond 50 min prior to that hour reading. Similarly, if a valve were to close 1 min after the hour, PHATR would wrongly have assumed that the valve was open for the next 59 min. This affected the bias (which increased or decreased, depending on the situation) but more importantly, this affected the model's ability to predict change (reflected in the correlation coefficient). The correlation coefficient decreased from 0.92 (step size,1 h) to 0.83 (step size, 4 h) to 0.70 (step size, 6 h). A step size of 10–30 min for heated model runs was suggested after a stability analysis was performed (Lamoureux, 2003).

6.3. Important energy vectors

Energy transfer mechanisms which were important to uncontrolled ponds were radiation heat transfer mechanisms (Fig. 5). The average importance of pond radiation, longwave sky radiation and solar radiation ranged between 15% and 54%, depending on the time of day and year. Solar and longwave sky radiation were, therefore the two most important influxes of energy for unheated ponds while pond radiation was the greatest source of heat loss. Evaporation also seemed to be important (range, 0–30%) although its average importance was small (4–13%) compared to the radiation heat transfer mechanisms. Air convection (average importance, 4–6%) and soil conduction (average importance, 2–6%) were not as important because the temperature difference which drove these heat transfer mechanisms was relatively small.

For heated ponds, in addition to radiation heat transfer mechanisms, warm water and discharged water represented important energy fluxes (Fig. 6). On average, warm water accounted for 19% of all energy transfer mechanisms while discharged water represented 15% of all energy fluxes. However, warm water could account for as much as 60% and discharged water as much as 44% of all transported energy.

Because the movement of water was an important energy vector (as important as 60% of all energy vectors), most errors in the model's output were likely attributed to estimations of flow. To fully understand the importance of bulk water movements to the energy balance, consider and compare the size of these energy vectors with solar radiation. On a clear sunny day in summer, solar power may be as high as 1000 W/m^2. For Pond 12, with an area of 280 m^2, this represented a heat transfer rate of 280 kW. To produce the same net energy flux with bulk flow, the flow rate must be 451 l/min (119 gpm), assuming the well water was 36 °C and the discharged water was 27 °C. This flow rate compares to a maximum flow rate of 2400 lpm (600 gpm) that the LSU ARS well is capable of providing. So for the purposes of comparison, warmwater flow can be considered as a night time substitute for sunlight. Flow was as important an energy transfer mechanism as sunlight and given flow fluctuations and measurement errors, PHATR's accuracy suffered.

Other transport mechanisms important to heated ponds were pond radiation (maximum relative importance, 50%), longwave sky radiation (maximum relative importance, 43%), solar radiation (maximum relative importance, 50%), evaporation (maximum relative importance, 41%) and air convection (maximum relative importance, 27%). Pond radiation, evaporation and convection are heat transfer mechanisms dependant on the temperature of the pond. In the spring, increases in pond temperature caused a greater temperature difference between the pond and the surrounding environment. Therefore, if the pond temperature were increased further, it is likely that these heat transfer mechanisms would increase in importance. Similarly, increases in wind speed will increase evaporation and convection. The extent of such increases would have to be addressed in a model sensitivity analysis. Heat transferred by the soil accounted for less than 6% of all energy movements.

7. Conclusions

A computer model, PHATR, was used to (1) predict heated and unheated pond temperatures and (2) determine the size of energy transfer mechanisms for such ponds. The model solved Eq. (1) using a Runge-Kutta 4th order numerical method. Comparisons between model runs and measured data indicated that PHATR over-predicted pond temperature (average bias for unheated ponds: 0.7 °C; for heated ponds: 2.4 °C). Flowrate fluctuations might have caused the larger biases in heated pond model runs. Using paddlewheel flowmeters connected to a data logger would provide a means of collecting more accurate instantaneous data. For both heated and unheated ponds, the model was good at predicting temperature changes ($r_{avg} = 0.94$ for unheated ponds, $r_{avg} = 0.87$ for heated ponds). The model's accuracy could be improved if the effects of sunlight on the energy balance were better understood (since there is a possible error of 6% associated with solar radiation). Such effects include light being reflected at the pond surface and the absorption and scattering of light in the pond water.

Major vectors of energy transfer for unheated ponds were found to be radiation heat transfer mechanisms. For heated ponds, bulk energy flow rates were also important. Surface convection and evaporation were important when there was no solar radiation or water flowing into the pond. The modeled heat conducted through the soil had a negative effect on PHATR's ability to predict change.

A sensitivity analysis would confirm the model's susceptibility to possible sources of error such as empirical equations and flow rate measurements. The analysis would also identify which environmental parameters have a greater impact on the model's output (Lamoureux et al., 2006). Identifying such parameters would be useful in recommending management practices for researchers and others using heated ponds. With respect to channel catfish, heating small (less than 2.6 ha) broodstock ponds would provide a significant benefit to fingerling producers, especially those involved in production of hybrids of channel catfish and blue catfish (*Ictalurus furcatus*) Indeed, commercial fingerling producers have begun to install and use geothermal wells to heat broodstock ponds in Mississippi (Lang et al., 2003).

PHATR must still be tested for ponds of different sizes and ponds located in different climates. For instance, evaporation could be much more important in a dryer climate and consequently, dryer air might negatively affect PHATR's accuracy. Larger ponds are also prone to greater absolute pond surface evaporation and convection and this in turn might affect the model's output.

PHATR can be a useful tool when designing temperature control devices and systems for outdoor earthen aquaculture ponds in the Southeastern United States. PHATR can be used to determine which energy vectors are important and consequently which ones need to be manipulated in order to conserve as much pond energy as possible.

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