Analysis and Forecasting of Seasonal Time Series

For analysis and forecasting of time series containing trend and seasonal variations a multiplicative-additive model considered is:

$\mathbf{Y_t = T_t \times S_t + E_t}$ { where $E_t \sim IIN(0, \sigma_E^{\,2})$ }.

An estimate of model is:

$\mathbf{Y_t^{\wedge} = T_t^{\wedge} \times S_t^{\wedge}}$.

The trend component T_t of this model is estimated by the simple linear regression model and the seasonal component S_t is estimated by using the Moving Average Model by taking the moving average period P equal to the seasonal period (the number of observations in a year) .

Example #1 (Analysis of Trivial time series using Moving Average Model)

Given below are time series recorded three times a year. These series are analyzed by using the above model . The trend values of this model are obtained using the regression line equation: $\mathbf{T_t^{\wedge}=27- 0.8t}$ and the seasonal variations are estimated through the three period moving average method.

Table #1.

Year	**Period**	**Time(t)**	$\mathbf{Y_t}$	**Tt^ =MA(3)**	$\mathbf{I_t=Y_t/T_t^{\wedge}}$		
2009	P_1	1	21	-----	----		
	P_2	2	30	75/3=25	1.20		
	P_3	3	24	72/3=24	1.00		
2010	P_1	4	18	72/3=24	0.75		
	P_2	5	30	75/3=25	1.20		
	P_3	6	27	72/3=24	1.13		
2011	P_1	7	15	75/3=25	0.60		
	P_2	8	33	57/3=19	1.74		**Seasonally Adjusted**
	P_3	9	9	-	-		**Forecasts**
				$\mathbf{T_t^{\wedge}}$		$\mathbf{S_t^{\wedge}}$	$\mathbf{f_{t+1}=T_t^{\wedge}xS_t^{\wedge}}$
2012	P_1	10	-	**19.0**	-	**0.649**	**19.0x0.649=12.3310**
	P_2	11	-	**18.2**	-	**1.327**	**18.2x1.327=24.1514**
	P_3	12	-	**17.4**	-	**1.024**	**17.4x1.024=17.8176**

Table #2 (**Estimation of seasonal variations (or Indices)** $\mathbf{S_t^{\wedge}}$ **using** I_t –values)

Year/Period	P_1	P_2	P_3	∑
2009	----	1.20	1.00	
2010	0.75	1.20	1.13	
2011	0.60	1.74	-----	
∑	1.35	4.14	2.13	
Averages: AVs	0.675	1.38	1.065	∑AVs= 3.12 → Adjustment Factor: AF= AVs/3 = **1.04**
$S_t^{\wedge}$=AVs/AF	**0.649**	**1.327**	**1.024**	$\sum \mathbf{S_t^{\wedge}=3.00}$ **(**=Seasonal/Moving Average Period)

Example #2 (Biannual Time Series)
Given below are data on sale of ice cream , recorded two times a year over the time period of 5 years. Let us analyze these data by considering the model: $\mathbf{Y_t=T_t\, xS_t + E_t}$ and using **two period moving average** (= seasonal period) and project the trend by using the regression line equation: $\mathbf{T_t^{\wedge}= 3.4+0.7429t.}$

Table #1

Year	**Season**	**Time (t)**	$\mathbf{Y_t}$	**MA (P=2)**	**Trend Centred MA=Tt^**	$I_t=Y_t/T_t^{\wedge}$	**Seasonal Indices**	**Seasonally Adjusted f'casts**
2009	S	1	**6**	----	-----	------		
				4				
	W	2	**2**		(4+4.5)/2=4.25	0.4706	---	---
				4.5				
2010	S	3	**7**		(4.5+5)/2=4.75	1.4737	---	---
				5				
	W	4	**3**		(5+7.5)/2=6.25	0.4800	---	---
				7.5				
2011	S	5	**12**		(7.5+9)/2=8.25	1.4545	---	---
				9				
	W	6	**6**		--------	-----	---	---
			----	-----	$\mathbf{T_t^{\wedge}}$	---	$S_t^{\wedge}$	$\mathbf{f_{t+1}=T_t^{\wedge}xS_t^{\wedge}}$
2012	**S**	**7**	---	-----	**8.6003**	**---**	1.5098	**12.9847**
	W	**8**	---	-----	**9.3432**	**---**	0.4902	**4.5600**

Table #2: Estimation of seasonal variations (or Indices) $\mathbf{S_t^{\wedge}}$ **using** I_t -values

Year	**Summer (S)**	**Winter (W)**	∑
2009	---------	0.4706	
2010	1.4737	0.4800	
2011	1.4545	---------	
∑	2. 9282	0.9506	
Averages (Av)	1.4641	0.4753	∑AVs=1.9394→ AF = AVs /2= **0.9697**
$\mathbf{S_t^{\wedge}}$**=AVs/AF**	1.5098	0.4902	∑ $S_t^{\wedge}$=2.0 (Seasonal/Moving Av. period)

Note that:
i)In seasonal forecasting the **Moving Average period P = Seasonal Period** (the number of times data are observed in a year). In our case, P=2 and sale figures are recorded two times a year. The moving /seasonal period in case of quarterly data is 4 and monthly data is 12.

ii) The **centralization** of the moving averages for alignment to the seasonal periods is required in case of even (P= 2,4,…) seasonal period only (not in case of odd seasonal/moving period; e.g. P=3,5,..) as the moving averages are already corresponding to the seasonal periods and not in between the periods.

Holt Winter Three Parameter Seasonal Model

For an observation at time t the model is written as:

$Y_t = (a_t + b_t)\ I_{t-L} + E_t$

Where at time t:
a is the level of the underlying process of the time series.
b is the growth
I is the seasonal index
L is length of seasonal cycle
E is a random error

Estimation of the model parameters

Level: $a_t = \alpha\ [\ Y_t / I_{t-L}\ ] + (1-\alpha)\ (a_{t-1} + b_{t-1})$

Growth: $b_t = \beta\ (a_t - a_{t-1}) + (1-\beta)\ b_{t-1}$

Seasonality: $I_t = \gamma\ (Y_t/a_t) + (1-\gamma\ )I_{t-1}$

k-steps ahead forecast is:

$f_{t+k} = (a_t + kb_t)\ I_{t-L+k}$