P7

$$= \psi_n^{*(0)} E_n^{*(0)}$$

substitute this value in the integral.

$$= \int \psi_n^{*(0)} (E_n^{*(0)} - E_n^{*(0)}) \psi_n^{(1)} d\tau$$

As we have $E_n^{*(0)} = E_n^{(0)}$ $\because$ energy can not be imaginary.

substitute in the relation.

$$\int \psi_n^{*(0)} (E_n^{(0)} - E_n^{(0)}) \psi_n^{(1)} d\tau = 0$$

Substitute this value in eq (4b).

$$0 = E_n^{(1)} - V_{nn}$$

$$\boxed{E_n^{(1)} = V_{nn}}$$

$$E_n^{(1)} = \int \psi_n^{*(0)} V \psi_n^{(0)} d\tau$$

So we have calculated the 1st order perturbation energy.

Calculation of 1st order correction to wave function:

Write the 1st order equation ~~an~~ and rearrange it.

$$H^{(0)} \psi_n^{(1)} + V \psi_n^{(0)} = E_n^{(1)} \psi_n^{(0)} + E_n^{(0)} \psi_n^{(1)}$$

Now re-arrange it

$$(H^{(0)} - E_n^{(0)}) \psi_n^{(1)} = E_n^{(1)} \psi_n^{(0)} - V \psi_n^{(0)} \quad \text{— (5)}$$

Now mlty this eqution by $\psi_m^{*(0)}$ on both sides and integrate w.r.t all coordinates

$$\int \psi_m^{*(0)} (H^{(0)} - E_n^{(0)}) \psi_n^{(1)} d\tau = \int \psi_m^{(0)*} E_n^{(1)} \psi_n^{(0)} d\tau - \int \psi_m^{(0)*} V \psi_n^{(0)} d\tau \quad (6)$$

take the 1st term of R.H.S and solve it.

$$\Rightarrow \int \psi_m^{(0)*} E_n^{(1)} \psi_n^{(0)} d\tau$$

take out energy of the integral

$$= E_n^{(1)} \int \psi_m^{(0)*} \psi_n^{(0)} d\tau$$

take the condition of orthogonality $m \neq n$ we take condition $m \neq n$ so we apply the condition of orthogonality. so our integral become zero.

$$\int \psi_m^{(0)*} \psi_n^{(0)} d\tau = 0$$

so our 1st term is zero. Now substitute this equality in eq (6)

$$\int \psi_m^{(0)*} (H^{(0)} - E_n^{(0)}) \psi_n^{(1)} d\tau = 0 - \int \psi_m^{(0)*} V \psi_n^{(0)} d\tau \quad (6a)$$

Now take the 2nd term of R.H.S and substitute in the form of symbol.

$$\text{ie} \int \psi_m^{(0)*} V \psi_n^{(0)} d\tau = V_{mn}$$

putting in eq (6a).

$$\int \psi_m^{(0)*}(H^{(0)} - E_n^{(0)})\psi_n^{(1)} d\tau = 0 - V_{mn} \quad \text{— 6b}$$

Now take L.H.S of eq and solve it.

$$\int \psi_m^{(0)}(H^{(0)} - E_n^{(0)})\psi_n^{(1)} d\tau$$

As $H^{(0)}$ can not operate on $\psi_n^{(1)}$, it can only operate on $\psi_n^{(0)*}$, so we take Ist term

$$\psi_m^{(0)*} H^{(0)} = H^{(0)} \psi_m^{(0)*}$$
$$= E_m^{(0)*} \psi_m^{(0)*}$$
$$= \psi_m^{(0)*} E_m^{(0)*}$$

Now putting in above integral.

$$\int \psi_m^{(0)*} (E_m^{*(0)} - E_n^{(0)}) \psi_n^{(1)} d\tau$$

put in eq 6b.

$$\int \psi_m^{(0)*} (E_m^{(0)*} - E_n^{(0)}) \psi_n^{(1)} d\tau = -V_{mn}$$

take out the energy term out side the integral.

$$= (E_m^{(0)*} - E_n^{(0)}) \int \psi_m^{(0)*} \psi_n^{(1)} d\tau = -V_{mn} \quad \text{— 7}$$

Divide on both sides by $(E_m^{(0)*} - E_n^{(0)})$

$$\int \psi_m^{(0)*} \psi_n^{(1)} d\tau = \frac{-V_{mn}}{(E_m^{(0)*} - E_n^{(0)})} \quad \text{— 7a}$$

we have $E_m^{(0)*} = E_m^{(0)}$,

$$\int \psi_m^{(0)*} \psi_n^{(1)} d\tau = - \frac{V_{mn}}{(E_m^{(0)} - E_n^{(0)})} \quad \text{—— (7b)}$$

As we have applied the condition of orthognality for function $\psi_m^{(0)}$. So we use an other state 'S' to express the wave function $\psi_n^{(1)}$.

Then $\psi_n^{(1)} = \Sigma C_{ns}^{(1)} \psi_s^{(0)}$

Now substitute this value in eq (7b).

$$\int \psi_m^{(0)*} \cdot \Sigma C_{ns}^{(1)} \psi_s^{(0)} d\tau = \frac{- V_{mn}}{(E_m^{(0)} - E_n^{(0)})}$$

Now take out cofficient out side of the integral.

$$\sum_s C_{ns}^{(1)} \int \psi_m^{(0)*} \psi_s^{(0)} d\tau = \frac{- V_{mn}}{(E_m^{(0)} - E_n^{(0)})}$$

substitute the integral in the form of kronenical delta.

$$\int \psi_m^{*(0)} \psi_s^{(0)} d\tau = \delta_{ms}$$

So

$$\sum_s C_{ns}^{(1)} \delta_{ms} = \frac{- V_{mn}}{(E_m^{(0)} - E_n^{(0)})} \quad \text{—— (8)}$$

Now apply the condition $s = m$

then $\delta_{mm} = 1$ by normalization condition.