1.4 The Compton Effect

The Nobel Prize in Physics, 1927: jointly-awarded to Arthur Holly Compton (figure 9),

for his discovery of the effect named after him.

$$\text{Compton} = \text{Scary}^2 \qquad (1.1)$$

Figure 9: Arthur Holly Compton (1892–1962): joint-winner of Nobel Prize for Physics in 1927.

Pre-1923 :x-rays scattered by electrons in matter were all though to have the same wavelength as that of the incident x-rays.

- Scattering of x-rays by matter was considered to be an elastic process – no energy is exchanged between the scattered x-rays and matter during the scattering.
- Such elastic scattering is known as **Thomson scattering**, after J J Thomson.

1923 :A H Compton carried out a careful study of the x-rays scattered by a thin layer of carbon (in the form of graphite) using the then recently developed Bragg x-ray diffractometer. He employed a beam of (essentially) monochromatic x-rays (figure 10).

- Compton found that the scattered x-rays had **two** components in the scattering direction (figure 11):
  - One component had a wavelength λ_0 equal to that of the incident radiation.

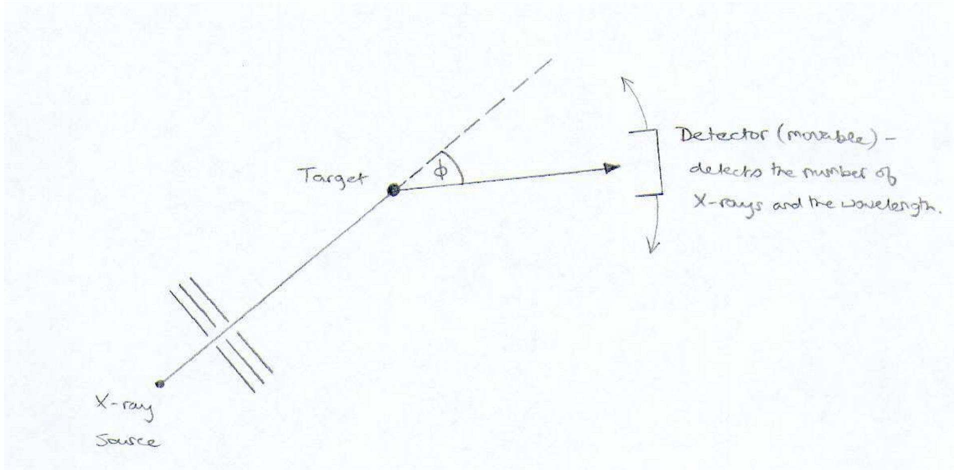

Figure 10: Basic schematic of Compton's experiment.

- Second component had a wavelength $\lambda = \lambda_0 + \Delta\lambda$.

- The apparent shift in wavelength, $\Delta\lambda$, is called the **Compton shift**.

- Compton found that

  - $\Delta\lambda$ varies with the scattering angle ϕ (see figure 11).
  - $\Delta\lambda$ increases rapidly at large scattering angles.
  - $\Delta\lambda$ is independent of the incident wavelength λ_0.
  - $\Delta\lambda$ is independent of the scattering material.

Classically: the carbon atoms in the graphite should oscillate at the frequency ν_0 of the incident radiation, and be re-radiated at the same frequency/wavelength.

Thus, the observed experimental results cannot be explained using classical physics.

Upon realising that classical physics cannot explain this effect, Compton embarked upon a more radical explanation based upon the (at the time) new quantum theory.

- Since the energy of an x-ray photon is very much larger than the binding energy of an atomic electron, the electron can be thought of as being "free".

- Compton assumed that x-rays could be treated as a stream of **photons**.

- The above experiment can thus be modeled as the scattering of photons by free electrons in the target material.

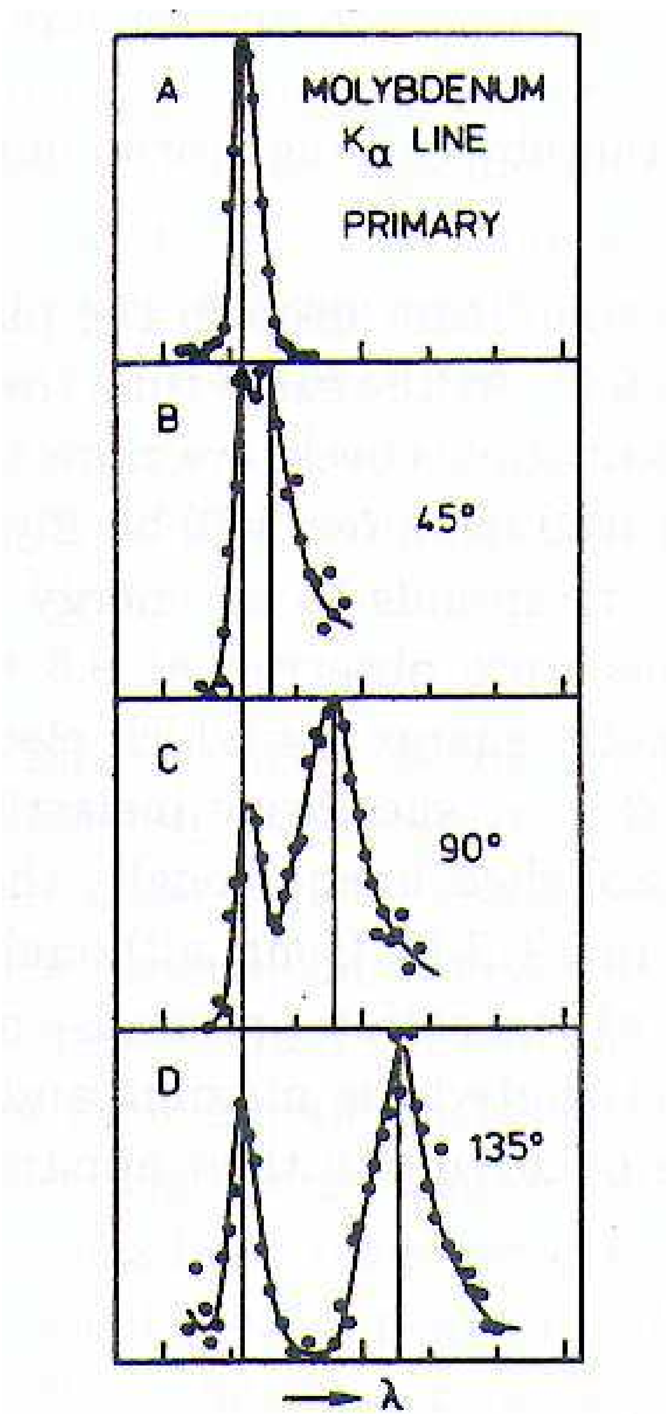

Figure 11: Compton's 1923 x-ray scattering results (from *Phys. Rev.* volume 21, page 483 (1923).

Comsider the pre- and post-collisional configuration of the scattering event (figure 12). Since the electrons have extremely small mass, and the energy associated with x-ray photons is very large, we need to employ a **relativistic** description of the scattering process.

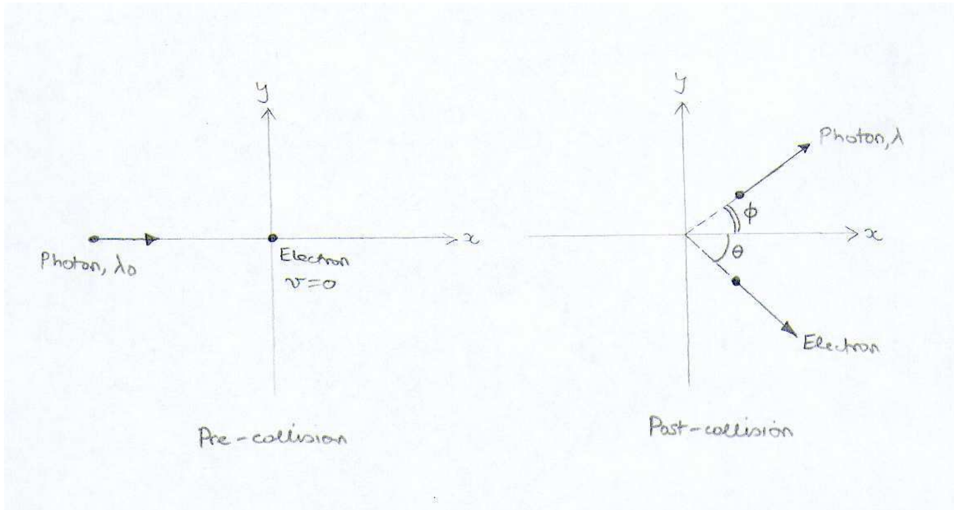

Figure 12: Basic dynamics of the Compton Effect.

Electron:

- Using the principle of equivalence of mass and energy (Einstein, 1911), the energy of an electron is given by

$$E = mc^2 = K + m_0c^2 \; . \tag{1.40}$$

Here,

  - m is the **relativistic mass** of the electron.
  - m_0 is the **rest-mass** of the electron.
  - K is the kinetic energy associated with the translational motion of the electron.

- According to special relativity, m and m_0 are related to each other by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \gamma m_0 \; , \tag{1.41}$$

where γ is the **Lorentz factor** and $v = |\underline{v}|$ is the speed of the electron.

- The linear momentum $\underline{p}$ of a moving electron is given by

$$\underline{p} = m\underline{v} = \gamma m_0 \underline{v} \; , \tag{1.42}$$

and its magnitude can be related to its **total (relativistic) energy**, E, by

$$p = mv = \frac{mc^2 v}{c^2} = \frac{Ev}{c^2} \; . \tag{1.43}$$

Photon:

- Equation (1.43) is the relation for a particle, such as an electron, which has a finite rest-mass. However, the final version of (1.43) does not explicitly contain the rest-mass of the electron.
- We can therefore obtain the relationship between the linear momentum of a photon (or any other particle with a zero rest-mass) simply by taking the limit that $m_0 \to 0$, or equivalently, $v \to c$; this gives

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \; . \tag{1.44}$$

Consider the pre- and post-collisional configurations outlined in figure 12: we note that photons carry both energy and linear momentum, so that we must consider both conservation laws simultaneously.

- **Conservation of energy**: equating total pre- and post-collisional energies, we have that

$$h\nu_0 + m_0c^2 = h\nu + mc^2 \; . \tag{1.45}$$

Using the expression $c = \nu\lambda$, (1.45) can be rewritten as

$$\frac{hc}{\lambda_0} + m_0c^2 = \frac{hc}{\lambda} + mc^2 \; , \tag{1.46}$$

or

$$\frac{h}{\lambda_0} - \frac{h}{\lambda} + m_0c = mc \; . \tag{1.47}$$

- **Conservation of Momentum**: equating the x- and y-components,

$$x - \text{component}: \qquad \frac{h\nu_0}{c} = \frac{h\nu}{c}\cos\phi + \gamma m_0 v\cos\theta \tag{1.48}$$

$$y - \text{component}: \qquad 0 = \frac{h\nu}{c}\sin\phi - \gamma m_0 v\sin\theta \; . \tag{1.49}$$

In terms of λ, (1.47) and (1.48) can be written as

$$\frac{h}{\lambda_0} - \frac{h}{\lambda}\cos\phi = \gamma m_0 v\cos\theta \; , \tag{1.50}$$

and

$$\frac{h}{\lambda}\sin\phi = \gamma m_0 v \sin\theta \ , \tag{1.51}$$

respectively.

Squaring and adding (1.50) and (1.51) yields the result

$$\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2\cos\phi}{\lambda_0\lambda} = \gamma^2 m_0^2 v^2 = \gamma^2 m_0^2 c^2 - m_0^2 c^2 \ . \tag{1.52}$$

Squaring (1.47) gives

$$\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda_0\lambda} + 2m_0 hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) + m_0^2 c^2 = \gamma m_0^2 c^2 \ . \tag{1.53}$$

Subtracting (1.52) from (1.53) gives the result

$$\frac{2h^2}{\lambda_0\lambda}(\cos\phi - 1) + 2m_0 hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) = 0 \ , \tag{1.54}$$

or equivalently,

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_0 c}\left(1 - \cos\phi\right) \ . \tag{1.55}$$

Using the trigonometric identity $\cos 2A = 1 - 2\sin^2 A$ with $2A = \phi$, this result can be written in the alternative form

$$\Delta\lambda = 2\frac{h}{m_0 c}\sin^2\frac{\phi}{2} \ . \tag{1.56}$$

- So (1.55) or (1.56) provides an expression for the Compton Shift $\Delta\lambda$.
- The expression $h/m_0 c$ is called the **Compton wavelength**.
- Employing the modern values of the physical constants:

  Planck constant: $h = 6.626075 \times 10^{-34}$ J s

  Electron rest-mass: $m_0 = 0.9109390 \times 10^{-30}$ kg

  Speed of light: $c = 2.997925 \times 10^8$ m s^{-1}

  we find that the Compton Shift may be expressed by

$$\Delta\lambda = 0.024263\left(1 - \cos\phi\right) \ , \tag{1.57}$$

  where we have expressed the Compton wavelength in units of Angstroms (1 $\mathring{A} = 10^{-10}$ m).

This calculation accounts for the shifted line. To explain the unshifted line, we need to understand that not all the scattering of x-rays is done by the free electrons.

- The **free electrons** recoil in the way we have explained above, and provide the shifted wavelength.
- The **valence/core electrons** do not recoil as above.
  - The electron mass in the above calculation would effectively be replaced by the mass of the atom in which the electron is located.
  - For such a large mass, the Compton Shift $\Delta\lambda$ for the scattered photon would be undetectable, i.e. $\Delta\lambda \approx 0$.
  - Therefore, it is the scattering of photons by the valence/core electron which leads to the unshifted (i.e. original) wavelength.