One Dimensional State Space

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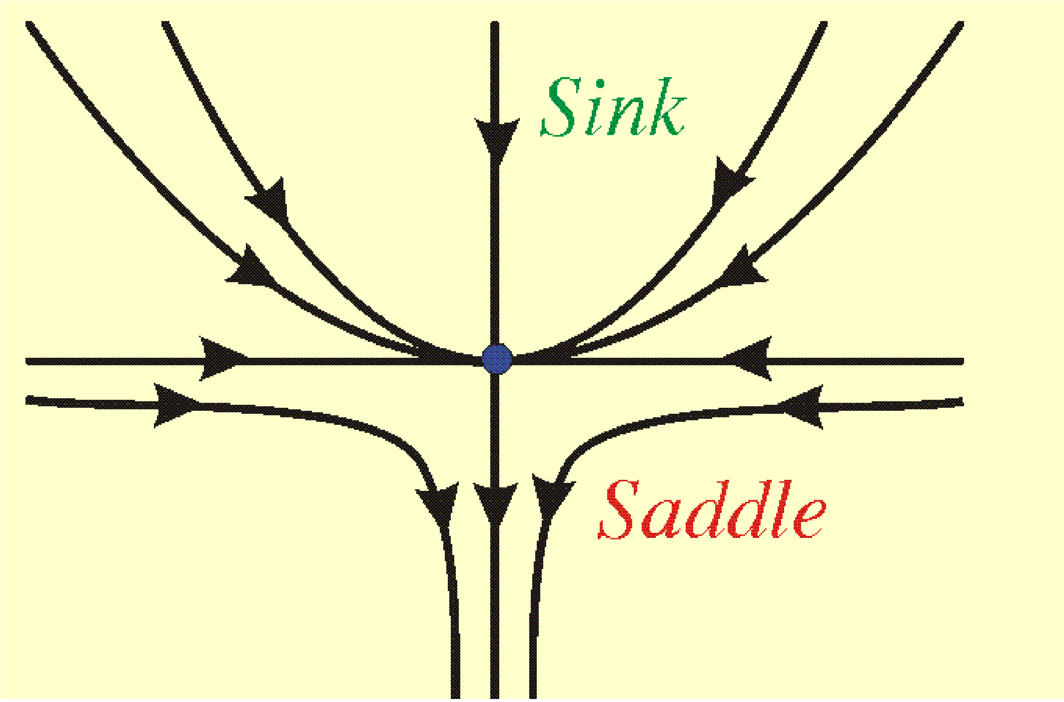
One Dimensional State Space

In one dimensional phase space there are three types of fixed points.

1. Nodes
2. Repellors
3. Saddle points



❑ Nodes

Nodes are fixed points that attract trajectories.

❑ Repellors

Repellors are fixed points that repels trajectories.

❑ Saddle points

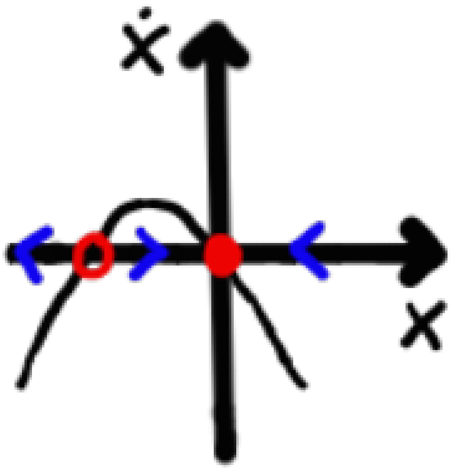
Saddle points are fixed points that attract the trajectories on one side and repel them on the other side.

Nodes (Attractors)

Let us consider a point x_0+x just right of fixed point x_0 as shown in the figure.
x_0 is the point on which trajectory is cutting the x-axis.
Since, x is +ive. Now if we move from left of the 'x_0' to 'x_0+x' the trajectory becomes more and more –ive. **Then the fixed point s known as attractor**.

If $f(x_0+x)$ is –ive then 'x' is –ive that means derivative of $f(x_0+x)$ is –ive.

$$\frac{df(x)}{dx}\Big|_{x=x_0} = \lambda < 0$$



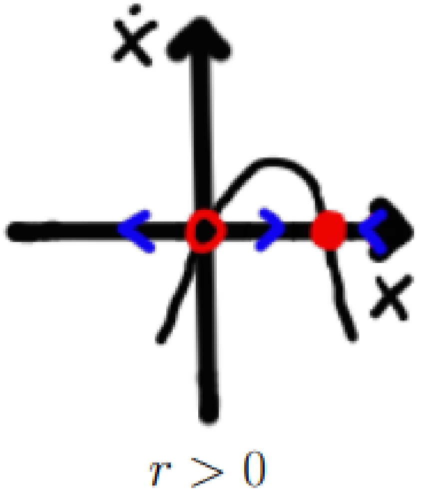
$r < 0$

The value of this derivative at fixed point is called the characteristics value or eigen value. So, $\lambda < 0$ for attractors.

Repellors

If **f(x_0+x)** is **+ive** then '**x**' is **+ive** that means if we move from left of the '**x_0**' to '**x**', the trajectory becomes more and more **+ive**. **Then the fixed point s known as repellors** . For a repellor, this can also be said that the values of the derivative of **f(x)** with respect to '**x**' is **+ive** at just right of fixed point.

$$\frac{df(x)}{dx}\Big|_{x=x_0} = \lambda > 0$$

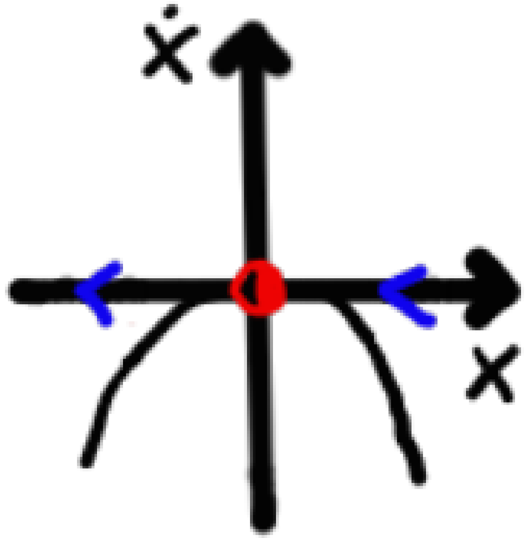


In figure the point where red circle is marked is known as **repellors**.

Saddle Points

A fixed points which attract the trajectory on one side and repel the trajectory on the other side, **then the fixed points is called Saddle points.**

$$\frac{df(x)}{dx}\bigg|_{x=x_0} = \lambda = 0$$



Here, the Eigen value is zero.
In figure the point where red circle is marked is known as **saddle points**.