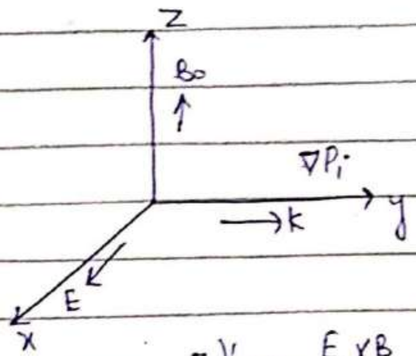
26-02-2019. Tuesday.

Magnetosonic Wave:-

It means sound wave in the presence/form of magnetic field.

- Sound waves in the presence of magnetic field is called magnetosonic waves.
- It is electromagnetic wave because "E & k" are ⊥ar to each other.
- "E" is along x-axis, "k" is along y-axis & B_0 is along z-axis.
- Ions plays important role in low frequency waves.
- ∇P_i is along the y-axis.
- Due to drift, compressions and rarefactions are ~~are~~ expected so, we take thermal pressure ∇P_i along y-axis.



$\because V_y = \frac{E \times B}{B^2}$

$\because V_{th}^2 = \frac{2K_BT}{m}$

Equation of motion for ions:-

$$m_i \cancel{n_{i0}} \frac{\partial V_{i1}}{\partial t} = \cancel{n_{i0}} e\left[E_1 + V_{i1} \times B_0\right] - \frac{\bar{\nabla} P_i}{n_{i0}}$$

$$m_i \frac{\partial V_{i1}}{\partial t} = e\left[E_1 + V_{i1} \times B_0\right] - \frac{\bar{\nabla} P_i}{n_{i0}}$$

X- component:-

$$\frac{\partial V_{ix}}{\partial t} = \frac{e}{m_i}\left(E_x + V_{iy} B_0\right)$$

$\because \frac{\nabla P_i}{n_{i0}} = 0$ because it is along y-axis.

$$V_{ix} = \frac{ie}{m_i \omega}\left(E_x + V_{iy} B_0\right) \longrightarrow (1)$$

Y- component:

$$\frac{\partial V_{iy}}{\partial t} = \frac{e}{m_i}\left(0 - V_{ix} B_0\right) - \frac{1}{m_i n_0} \gamma_i K_B T_i \nabla n_i$$

$$V_{iy} = \frac{-i e B_0}{m_i \omega} V_{ix} + \frac{\gamma_i K_B T_i}{m_i n_{i0}} \frac{\cancel{i} k}{\cancel{i} \omega} n_{i1}$$

$$V_{iy} = -\frac{i\omega_{ci}}{\omega} V_{ix} + \frac{\gamma_i k_B T_i}{m_i} \frac{k}{\omega} \frac{n_{i1}}{n_{i0}} \quad \text{——} (2)$$

Linearized Continuity Equation:

$$\frac{\partial n_{i1}}{\partial t} + n_0 \nabla \cdot V_{i1} = 0$$

$$(-i\omega n_{i1}) + n_0 i k V_{iy} = 0$$

$$\frac{n_{i1}}{n_0} = \frac{k}{\omega} V_{iy} \quad \text{——} (3)$$

Substituting it in eq. (2)

$$V_{iy} = -\frac{i\omega_{ci}}{\omega} V_{ix} + \frac{\gamma_i k_B T_i}{m_i} \frac{k}{\omega} \left(\frac{k}{\omega} V_{iy} \right)$$

$$V_{iy} = -\frac{i\omega_{ci}}{\omega} V_{ix} + \frac{\gamma_i k_B T_i}{m_i} \frac{k^2}{\omega^2} V_{iy}$$

Suppose $\frac{\gamma_i k_B T_i}{m_i} \frac{k^2}{\omega^2} = A_i$

$$V_{iy} = -\frac{i\omega_{ci}}{\omega} V_{ix} + A_i V_{iy} \quad \text{——} (4)$$

$$V_{iy} (1 - A_i) = -\frac{i\omega_{ci}}{\omega} V_{ix}$$

$$V_{iy} = -\frac{i\omega_{ci}}{\omega} (1 - A_i)^{-1} V_{ix} \quad \text{——} (5)$$

Substituting eq. (5) in eq. (1)

$$V_{ix} = \frac{ie}{m_i \omega} E_x + \frac{ieB_0}{m_i \omega} \left(-\frac{i\omega_{ci}}{\omega} (1 - A_i)^{-1} V_{ix} \right)$$

$$V_{ix} = \frac{ie}{m_i \omega} E_x + \frac{i\omega_{ci}}{\omega} \left(-\frac{i\omega_{ci}}{\omega} (1 - A_i)^{-1} V_{ix} \right)$$

$$V_{ix} = \frac{ie}{m_i \omega} E_x + \frac{\omega_{ci}^2}{\omega^2} (1 - A_i)^{-1} V_{ix}$$

$$\left(1 - \frac{\omega_{ci}^2}{\omega^2} (1 - A_i)^{-1} \right) V_{ix} = \frac{ieE_x}{m_i \omega}$$

$$V_{ix} = \frac{ie}{m_i \omega} \left(1 - \frac{\omega_{ci}^2}{\omega^2} (1 - A_i)^{-1} \right)^{-1} E_x$$

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$$V_{ix} = \frac{ie}{m_i\omega}\left(1 - \frac{\omega_{ci}^2}{\omega^2(1-A_i)}\right)^{-1} E_x \quad \text{---}(6)$$

Similarly for electrons,

$$V_{ex} = -\frac{ie}{m_e\omega}\left(1 - \frac{\omega_{ce}^2}{\omega^2(1-A_e)}\right)^{-1} E_x \quad \text{---}(7)$$

By using Maxwell's Equations, we have

$$(\omega^2 - c^2k^2)E_x = -i4\pi\omega n_0 e\,(V_{ix} - V_{ex})$$

Substituting the values of V_{ix} & V_{ex} from eq.(6) & eq.(7) in above equation.

$$(\omega^2 - c^2k^2)\cancel{E_x} = -i4\pi\omega n_0 e\left[\frac{ie}{m_i\omega}\frac{\omega^2(1-A_i)}{\omega^2(1-A_i)-\omega_{ci}^2} + \frac{ie}{m_e\omega}\frac{\omega^2(1-Ae)}{\omega^2(1-Ae)-\omega_{ce}^2}\right]\cancel{E_x}$$

$$\omega^2 - c^2k^2 = \omega_{pi}^2\frac{\omega^2(1-Ai)}{\omega^2(1-Ai)-\omega_{ci}^2} + \omega_{pe}^2\frac{\omega^2(1-Ae)}{\omega^2(1-Ae)-\omega_{ce}^2} \quad \text{---}(8)$$

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As $A_i = \frac{\gamma_i K_B T_i}{m_i}\frac{k^2}{\omega^2}$

& $A_e = \frac{\gamma_e K T_e}{m_e}\frac{k^2}{\omega^2}$

Since these are low frequency waves, the approximations of low frequency waves are

i) $\omega^2 << \omega_{ci}^2$

$\omega^2 << \omega_{ce}^2$

ii) $\omega^2 << k^2 V_{the}^2$

iii) $V_{the}^2 > V_{thi}^2$

Then eq.(8) becomes,

$$\omega^2 - c^2k^2 = \omega_{pi}^2\frac{(\omega^2\,(1-Ai)}{-\omega_{ci}^2} + \omega_{pe}^2\frac{(-\omega^2 A_e)}{-\omega_{ce}^2}$$

$$\omega^2 - c^2k^2 = \omega_{pi}^2 \frac{\omega^2(1-A_i)}{-\omega_{ci}^2} + \omega_{pe}^2 \frac{(-\omega^2 A_e)}{-\omega_{ce}^2} \quad \because \omega^2(1-A_e) = \omega^2 - \omega^2 \frac{\gamma_e K T_e}{m_e} \cdot \frac{k^2}{\omega^2}$$

$$\because \omega^2(1-A_e) = \frac{\omega^2 \gamma_e K T_e k^2}{m_e}$$

$$\omega^2 - c^2k^2 = -\frac{4\pi n_0 e^2}{\cancel{m_i}} \frac{\omega^2(1-A_i)}{\frac{e^2B_0^2}{m_i^2}} + \frac{4\pi n_0 e^2}{\cancel{m_e}} \frac{\omega^2 A_e}{\frac{e^2 B_0^2}{m_e^2}}$$

$$\omega^2 - c^2k^2 = -\frac{4\pi n_0 m_i}{B_0^2}\omega^2(1-A_i) + \frac{4\pi n_0 m_e}{B_0^2}\omega^2 A_e$$

$$\omega^2 - c^2k^2 = -\frac{4\pi n_0 m_i}{B_0^2}\omega^2\left[1 - \frac{\gamma_i K T_i}{m_i}\frac{k^2}{\omega^2}\right] + \frac{4\pi n_0 m_e}{B_0^2}\omega^2\left[\frac{\gamma_e K T_e}{m_e}\frac{k^2}{\omega^2}\right]$$

$$\omega^2 - c^2k^2 = -\frac{4\pi n_0 m_i}{B_0^2}\omega^2 + \frac{4\pi n_0 m_i \cancel{\omega^2}}{B_0^2}\frac{\gamma_i K T_i}{m_i}\frac{k^2}{\cancel{\omega^2}} + \frac{4\pi n_0 \cancel{m_e}}{B_0^2}\frac{\gamma_e K T_e}{\cancel{m_e}}\frac{k^2}{\cancel{\omega^2}}\cancel{\omega^2}$$

$$\omega^2 - c^2k^2 = -\frac{4\pi n_0 m_i \omega^2}{B_0^2} + \frac{4\pi n_0 m_i}{B_0^2}\frac{\gamma_i K T_i}{m_i}k^2 + \frac{4\pi n_0}{B_0^2}\frac{\gamma_e K T_e m_i k^2}{m_i}$$

$$\omega^2 - c^2k^2 = -\frac{4\pi \rho_i}{B_0^2}\omega^2 + \frac{4\pi\rho}{B_0^2}k^2\left(\frac{\gamma_i K T_i}{m_i} + \frac{\gamma_e K T_e}{m_i}\right) \quad \because n_0 m_i = \rho_i$$

Multiplying & dividing by "c^2" on R.H.S,

$$\omega^2 - c^2k^2 = -\frac{4\pi\rho_i c^2}{B_0^2 c^2}\omega^2 + \frac{4\pi\rho}{B_0^2}\frac{c^2 k^2}{c^2}\left(\frac{\gamma_i K T_i}{m_i} + \frac{\gamma_e K T_e}{m_i}\right)$$

$$\omega^2 - c^2k^2 = -\frac{c^2\omega^2}{V_A^2} + \frac{c^2k^2}{V_A^2}\cdot V_s^2$$

$$\omega^2 + \frac{c^2\omega^2}{V_A^2} = +c^2k^2 + \frac{c^2k^2}{V_A^2}\cdot V_s^2$$

$$\omega^2\left(1 + \frac{c^2}{V_A^2}\right) = k^2\left(c^2 - \frac{c^2}{V_A^2}V_s^2\right)$$

$$\omega^2\left(1 + \frac{c^2}{V_A^2}\right) = c^2k^2\left(1 - \frac{V_s^2}{V_A^2}\right)$$

$$\frac{\omega^2}{k^2} = \frac{c^2\left(1 - \frac{V_s^2}{V_A^2}\right)}{1 + \frac{c^2}{V_A^2}}$$

$$\frac{\omega^2}{k^2} = \frac{c^2 (V_A^2 + V_s^2) / \cancel{V_A^2}}{\frac{V_A^2 + c^2}{\cancel{V_A^2}}}$$

$$\frac{\omega^2}{k^2} = \frac{c^2 (V_A^2 + V_s^2)}{(V_A^2 + c^2)}$$

This is the dispersion relation for magnetosonic wave.

If there is no magnetic field then V_A become zero so, we again reach to the dispersion relation of ion sound waves.

Phase Velocity:-

It is more than the Alfven velocity so magnetosonic waves are also known as the fast hydrodynamic waves.