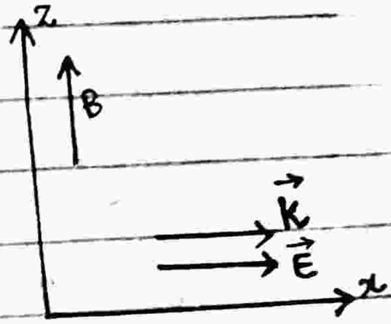
Electrostatic Electron Oscillations $\perp$r to B or Upper Hybrid Waves:-

Upper hybrid waves also called high frequency waves. We shall assume that the ions are too massive to move at the frequencies involved & form a fixed, uniform background of +ive charge. We shall also neglect thermal motions & set $KT_e = 0$

thus from Lorentz eq. of motion for ēs (linearized),

$$m_e n_{0e} \frac{\partial v_{e1}}{\partial t} = -n_{0e} e\left[\vec{E}_1 + (V_1 \times B_0)\right]$$



Applying character,

$$\frac{\partial V_1 e^{i(kx-\omega t)}}{\partial t} = -\frac{e}{m}\left[\vec{E}_1 + (V_1 \times B_0)\right] e^{i(kx-\omega t)}$$

$$-i\omega V_1 e^{i(kx-\omega t)} = -\frac{e}{m}\left[\vec{E}_1 + (V_1 \times B_0)\right] e^{i(kx-\omega t)}$$

∵ Electrostatic wave bcoz E & K are ‖.

$$i\omega V_1 = \frac{e}{m}\left[\vec{E}_1 + (V_1 \times B_0)\right]$$

$$V_1 = \frac{-ie}{m\omega}\left[\vec{E}_1 + (V_1 \times B_0)\right]$$

write eq. for x-component:

$$V_{1x} = \frac{-ie}{m\omega}\left(E_{1x} + V_{1y}B_0\right) \quad \text{——— (1)}$$

For y-component:

$$V_{1y} = \frac{-ie}{m\omega}\left(E_{1y} - V_{1x}B_0\right) \qquad \because E_y = 0$$

$$= \frac{-ie}{m\omega}\left(0 - V_{1x}B_0\right)$$

$$V_{1y} = \frac{ieB_0}{m\omega} V_{1x} \qquad \because \frac{eB_0}{m} = \omega_c$$

$$V_{1y} = \frac{i\omega_c}{\omega} V_{1x} \quad \text{——— (2)}$$

Putting eq. (2) in eq. (1),

$$V_{1x} = \frac{-ie}{m\omega} E_{1x} - \frac{ieB_0}{m\omega} \cdot \left(\frac{i\omega_c}{\omega}\right) V_{1x}$$

$$V_{1x} = \frac{-ie}{m\omega} E_{1x} + \frac{\omega_c}{\omega} \cdot \frac{\omega_c}{\omega} V_{1x}$$

$$V_{1x} = \frac{-ie}{m\omega} E_{1x} + \frac{\omega_c^2}{\omega^2} V_{1x}$$

$$V_{1x} - \frac{\omega_c^2}{\omega^2} V_{1x} = \frac{-ie}{m\omega} E_{1x}$$

$$V_{1x}\left(1 - \frac{\omega_c^2}{\omega^2}\right) = \frac{-ie}{m\omega} E_{1x}$$

$$V_{1x} = \frac{\frac{-ie}{m\omega} E_{1x}}{1 - \frac{\omega_c^2}{\omega^2}} \quad \text{——— (3)}$$

Now, write the linearized Continuity eq.:

$$\frac{\partial n_1}{\partial t} + n_0(\nabla \cdot V_1) = 0$$

Applying characters,

$$\frac{\partial n_1 e^{i(kx-\omega t)}}{\partial t} + n_0 \frac{\partial V_1 e^{i(kx-\omega t)}}{\partial x} = 0$$

$$-i\omega n_1 e^{i(kx-\omega t)} + n_0 ik V_{1x} e^{i(kx-\omega t)} = 0$$

$$e^{i(kx-\omega t)} (-i\omega n_1 + n_0 ik V_{1x}) = 0$$

$$-i\omega n_1 + n_0 ik V_{1x} = 0$$

$$n_0 ik V_{1x} = i\omega n_1$$

$$\frac{k}{\omega} = \frac{n_1}{n_0 V_{1x}}$$

$$V_{1x} = \frac{\omega}{k} \frac{n_1}{n_0} \quad \text{———} (4)$$

Comparing eq. (3) & (4),

$$\frac{\omega}{k} \frac{n_1}{n_0} = \frac{-ie}{m\omega} E_{1x} \Big/ 1 - \frac{\omega_c^2}{\omega^2} \quad \text{———} (5)$$

Now, write the linearized Poisson's eq.,

$$\nabla \cdot E_1 = -4\pi e n_{e1} \qquad \because \nabla \cdot E = 4\pi e (\overset{(0)}{n_i} - n_e)$$

Applying character,

$$\frac{\partial E_{1x} e^{i(kx-\omega t)}}{\partial x} = -4\pi e n_1 e^{i(kx-\omega t)}$$

$$ikE_{1x} e^{i(kx-\omega t)} = -4\pi e n_1 e^{i(kx-\omega t)}$$

$$ikE_{1x} = -4\pi e n_1$$

$$E_{1x} = +\frac{i4\pi e n_1}{k} \quad \text{———} (6)$$

Putting eq. (6) in eq. (5),

$$\frac{\omega}{k} \frac{n_1}{n_0} = \frac{-ie}{m\omega} \cdot \left(\frac{i4\pi e n_1}{k}\right) \Big/ 1 - \frac{\omega_c^2}{\omega^2}$$

$$\omega^2 = \frac{4\pi n_0 e^2}{m} \Big/ 1 - \frac{\omega_c^2}{\omega^2}$$

$$\omega^2 = \frac{4\pi n_0 e^2}{m} \Big/ \frac{\omega^2 - \omega_c^2}{\omega^2}$$

∵ Hybrid means mixture of two things.

$$\omega^2 = \omega_{pe}^2 \Big/ \frac{\omega^2 - \omega_c^2}{\omega^2}$$

∴ $\omega_{pe}^2 = \frac{4\pi n_0 e^2}{m}$

$$\omega^2 = \frac{\omega_{pe}^2 \cdot \omega^2}{\omega^2 - \omega_c^2}$$

$$\frac{(\omega^2 - \omega_c^2)\cancel{\omega^2}}{\cancel{\omega^2}} = \omega_{pe}^2$$

$$\omega_{pe}^2 = \omega^2 - \omega_c^2$$

$$\omega^2 = \omega_{pe}^2 + \omega_c^2$$

This is the dispersion relation of upper hybrid wave. & it is represented by a symbol ω_h which is the upper hybrid frequency. The group velocity is again zero as long as thermal motions are neglected. also phase velocity is zero because no k is present. This is the mixture of Langmuir frequency & cyclotion frequency. Also we can say this is a mixture of electric field & magnetic field. Two frequencies are added that's why high frequency wave or also called upper hybrid waves.