Non-Uniform E Field :-

Now, we let the magnetic field is uniform & the electric field be non-uniform. For simplicity we assume E to be in the x-direction & to vary sinusoidally in the x-direction as shown in Figure 1.

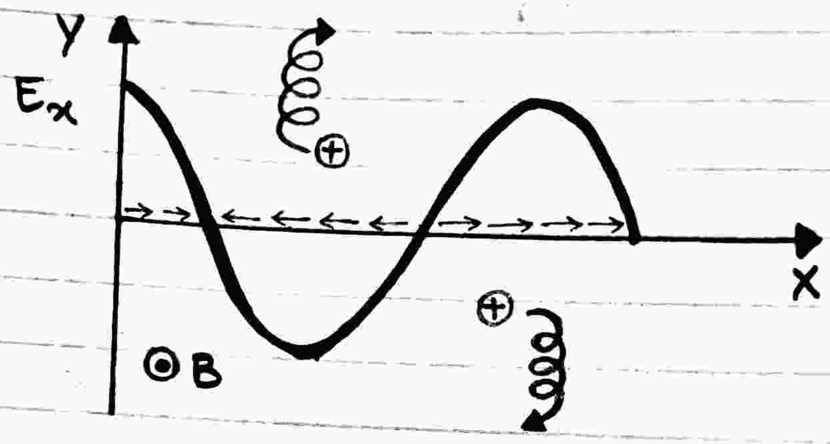

Figure 1

If body is oscillating it means it is not uniform because the body changes its phase & direction.

So,

electric field variation along x-direction can be written as:

$$E_{(x)} = E_o \cos(Kx)\,\hat{x} \quad \text{———} \quad (1)$$

In practice, a charge distribution can arise in a plasma during a wave motion. The eq. of motion is:

$$m\frac{d\vec{v}}{dt} = q\left(\vec{E}_x + v \times \vec{B}\right) \quad \text{———} \quad (2)$$

Now,

lets write the components,

x- component :-

$$\dot{V}_x = \frac{q}{m}(E_x + V_y B_0)$$

$$\dot{V}_x = \frac{q}{m}E_x + \frac{qB_0}{m}V_y$$

$\because \frac{qB}{m} = \omega_c$

$$\dot{V}_x = \frac{q}{m}E_x + \omega_c V_y \quad \text{------} (3)$$

y- component :-

$$\dot{V}_y = \frac{q}{m}(0 - V_x B_0)$$

$$\dot{V}_y = -\omega_c V_x \quad \text{------} (4)$$

Now, diff eq. (3) w.r.t time

$$\ddot{V}_x = \frac{q}{m}\dot{E}_x + \omega_c \dot{V}_y$$

$$\dot{E}_x = \frac{dE_x}{dt} = 0$$

because,

we are not concerned with non- uniform electric field w.r.t time.

So,

$$\ddot{V}_x = \omega_c \dot{V}_y \quad \text{------} (5)$$

put value of $\dot{V}_y$ from eq. (4) in eq. (5),

$$\ddot{V}_x = -\omega_c^2 V_x \quad \text{------} (6)$$

diff eq. (4) w.r.t time,

$$\ddot{V}_y = -\omega_c \dot{V}_x \quad \text{——} \quad (7)$$

put value of $\dot{V}_x$ from eq. (3) in eq. (7),

$$\ddot{V}_y = -\omega_c \left(\frac{q}{m} E_x + \omega_c V_y \right)$$

xing & ÷ing 1st term of above eq. by B_o,

$$\ddot{V}_y = -\omega_c \left(\frac{qB}{m} \frac{E_x}{B_o} + \omega_c V_y \right)$$

$$\ddot{V}_y = -\omega_c^2 V_y - \omega_c^2 \frac{E_x}{B_o} \quad \text{——} \quad (8).$$

Put value of E_x from eq. (1) in eq. (8),

$$\ddot{V}_y = -\omega_c^2 V_y - \omega_c^2 \frac{E_o}{B_o} \cos(kx) \quad \text{——} \quad (9)$$

In a uniform magnetic field the effect of Larmour radius is there so,

$$x = x_o + r_L \sin \omega_c t$$

put this value in eq. (9),

$$\ddot{V}_y = -\omega_c^2 V_y - \omega_c^2 \frac{E_o}{B_o} \cos k (x_o + r_L \sin \omega_c t) \quad \text{——} \quad (10)$$

As we are interested in finding an expression for V_E, we take out the gyratory motion by taking average over the cycle. Also, in eq. (10)

The oscillating term $\ddot{v}_y$ clearly averages to zero & we have,

$$\ddot{v}_y = 0 = -\omega_c^2 v_y - \omega_c^2 \frac{E_0}{B_0} \cos k(x_0 + r_L \sin\omega_c t)$$

$$\omega_c^2 v_y = -\omega_c^2 \frac{E_0}{B_0} \cos k(x_0 + r_L \sin\omega_c t) \qquad \text{(11)}$$

$$\cos k(x_0 + r_L \sin\omega_c t) = \cos kx_0 \cos(kr_L \sin\omega_c t) - \sin kx_0 \sin(kr_L \sin\omega_c t)$$

Also,

$$\cos\epsilon = 1 - \frac{1}{2}\epsilon^2 + \ldots$$

$$\sin\epsilon = \epsilon$$

putting values,

$$\cos k(x_0 + r_L \sin\omega_c t) = \cos kx_0 \left[1 - \frac{1}{2}(k^2 r_L^2 \sin^2\omega_c t) - \sin kx_0 \, kr_L \sin\omega_c t\right]$$

As we know,

$$\langle \sin^2\omega_c t \rangle = 1/2 \quad \& \quad \langle \sin\omega_c t \rangle = 0$$

$$\langle \cos^2\omega_c t \rangle = 1/2 \quad \& \quad \langle \cos\omega_c t \rangle = 0$$

Then by putting values of averages...

$$\cos k(x_0 + r_L \sin\omega_c t) = \cos kx_0 \left(1 - \frac{1}{4} k^2 r_L^2\right) - 0$$

$$= \cos kx_0 \left(1 - \frac{1}{4} k^2 r_L^2\right)$$

Put this value in eq. (11),

$$\omega_c^2 v_y = -\omega_c^2 \frac{E_0}{B_0} \cos kx_0 \left(1 - \frac{1}{4} k^2 r_L^2\right)$$

$$V_y = -\frac{E_o}{B_o} \cos kx_o \left(1 - \frac{1}{4} k^2 r_\ell^2\right)$$

$$V_y = -\frac{E_x}{B_o}\left(1 - \frac{1}{4} k^2 r_\ell^2\right)$$

Thus, the usual **E×B** drift is modified by the imhomogenity to read,

$$V_E = \frac{E \times B}{B^2}\left(1 - \frac{1}{4} k^2 r_\ell^2\right) \quad \text{—— (12)}$$

If we say that electric field is not oscillating then "$k=0$", then we have the usual "**E×B**" drift but -ive sign shows that this factor i.e, $-\frac{1}{4}k^2 r_\ell^2$ reduces the drift velocity.

It means, oscillating electric field is reduces the usual electric field drift.

Now,

$$E = E_o e^{i(k \cdot x)}$$

$$\frac{dE}{dx} = E_o ik e^{i(k\cdot x)}$$

$$\frac{d^2E}{dx^2} = -E_o k^2 e^{i(k\cdot x)}$$

$$\frac{d^2E}{dx^2} = -k^2 E_o e^{i(k\cdot x)}$$

$$\frac{d^2E}{dx^2} = -k^2 E$$

$$\nabla_x^2 = -k^2$$

put this value in eq. (12),

$$V_E = \frac{E \times B}{B^2}\left(1 + \frac{1}{4} \nabla^2 r_\ell^2\right)$$

As r_L is much larger for ions than for $\bar{e}s$, v_E is no longer independent of species.

If a density clumps occurs in a plasma, an electric field can cause the ions & $\bar{e}s$ to separate, generating another electric field. If there is a feedback mechanism that causes the second electric field to enhance the first one, **E** grows indefinately & the plasma is unstable. Such an instability is called the drift instability.

The grad-B drift is also a finite-Larmor-radius effect & also causes charges to separate.