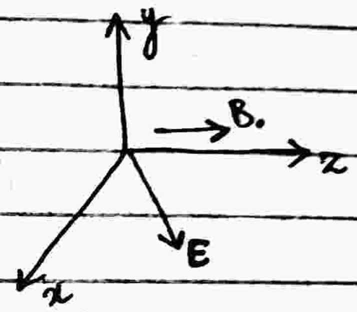
Finite E

If we allow an electric field to be present, the motion will be found to be the sum of two motions:
(i) the usual circular Larmor gyration plus
(ii) a drift of the guiding centre
We may choose E to lie in the x-z plane so that $E_y = 0$
If we have single plasma particle & we apply electric field on it along x-axis & magnetic field along z-axis then from Lorentz eq. of motion,

$$F = q(E + V \times B) \quad \text{—} \quad (1)$$



Resolve eq. (1) into x-component,

$$F = ma = q(E_x + V_y B)$$

$$m\frac{dV_x}{dt} = q(E_x + V_y B)$$

$$\dot{V}_x = \frac{q}{m}(E_x + V_y B) \quad \text{—} \quad (2)$$

$$V \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_x & V_y & V_z \\ 0 & 0 & B \end{vmatrix}$$

Now resolve eq. (1) into y-component,

$$F = ma = q(E_y - V_x B)$$

$$m\frac{dV_y}{dt} = q(E_y - V_x B)$$

Since,

there is no electric field along y-axis so, $E_y = 0$, above eq. becomes,

$$\dot{V_y} = \frac{q}{m}(0 - V_x B)$$

$$\dot{V_y} = -\frac{qB}{m} V_x$$

$$\dot{V_y} = -\omega_c V_x \quad \text{———} \quad (3)$$

Now resolve along z-axis,

$$F = ma = q(E_z + 0)$$

$$m\frac{dV_z}{dt} = qE_z$$

$$\frac{dV_z}{dt} = \frac{q}{m} E_z$$

Integrating b/s, w.r.t 't'

$$V_z = \frac{q}{m} E_z t + c$$

Applying boundary conditions,

$t = 0$ & assuming $V_z = V_{z_0}$

So,

$$V_z = \frac{q}{m} E_z t + V_{z_0}$$

It means that electric field is accelerating the particles.

diff eq. ② w.r.t time,

$$\ddot{V}_x = \frac{q}{m}(\dot{E}_x + \dot{V}_y B)$$

$$= \frac{q}{m}(0 + \dot{V}_y B)$$

∵ $\dot{E}_x = 0$ because electric field is not changing, it is uniform i.e, not changing with time so, $\frac{dE}{dt} = 0$

$$\ddot{V}_x = \frac{qB}{m}\dot{V}_y$$

$$\ddot{V}_x = \omega_c \dot{V}_y \quad \text{—— ④}$$

put value of $\dot{V}_y$ from eq ③ in eq. ④,

$$\ddot{V}_x = -\omega_c^2 V_x \quad \text{—— ⑤}$$

Taking derivative of eq. ③

$$\ddot{V}_y = -\omega_c \dot{V}_x \quad \text{—— ⑥}$$

put value from ② of $\dot{V}_x$ in eq. ⑥,

$$\ddot{V}_y = -\omega_c \cdot \frac{q}{m}(E_x + V_y B)$$

xing & ÷ing by B,

$$\ddot{V}_y = -\omega_c \cdot \frac{qB}{m}\left(\frac{E_x}{B} + V_y\right)$$

$$\ddot{V}_y = -\omega_c^2\left(\frac{E_x}{B} + V_y\right) \quad \text{—— ⑦}$$

or,

$$\frac{d^2 V_y}{dt^2} = -\omega_c^2\left(\frac{Ex}{B} + V_y\right)$$

Now,

$$\frac{d^2}{dt^2}\left(\frac{Ex}{B} + V_y\right) = -\omega_c^2\left(\frac{Ex}{B} + V_y\right) \quad (8)$$

↓

This is a constant, the derivative of constant is always zero (0), just we add $\frac{Ex}{B}$ here only for balancing of equation.

Soln. of eq. (8) is,

$$V_y = iV_\perp e^{i\omega_c t} - \frac{Ex}{B} \quad (9)$$

→ it has dimensions of velocity.

$\frac{Ex}{B}$ is basically a velocity that drift the particle along -ive y-axis.

Let's find out the dimension of above eq.
As we know exponent has no dimension
so, we are concerned with $\frac{Ex}{B}$
we know,

$$F = qE$$

$$E = \frac{F}{q}$$

also,

$$F = qVB$$

$$B = \frac{F}{qV}$$

By putting values,

$$\frac{Ex}{B} = F/q \Big/ F/qV = \frac{1/q}{1/qV} = 1/q \times qV$$

So, we are left with

$$\frac{E_x}{B} = V$$

Hence, it is clear that $V_\perp$ is the drift velocity & it is not along x & z axis. It means the drift velocity is $\perp r$ to (y-axis). (-ive y-axis).

Now, we will find the vector form of eq. (9),
as we know,

$$F = q(E + V \times B)$$

Suppose,

$$F = 0$$
$$q(E + V \times B) = 0$$
$$q \neq 0$$

∵ q is the charge & never equals to zero.

So,

$$\vec{E} + \vec{V} \times \vec{B} = 0$$
$$\vec{E} = -\vec{V} \times \vec{B}$$

Lets take the cross product of B on b/s,

$$\vec{E} \times \vec{B} = -(\vec{V} \times \vec{B}) \times \vec{B}$$
$$= \vec{B} \times (\vec{V} \times \vec{B}) \qquad \because A \times B = -B \times A$$

By applying identity

$$\because a \times (b \times c) = ab^2 - b(ab)$$

$$\vec{E} \times \vec{B} = \vec{V}B^2 - \vec{B}(\vec{V} \cdot \vec{B})$$

V is always $\perp r$ to B, it means $V \cdot B = 0$
because $V \cdot B$ is a scalar product.

$$\vec{E} \times \vec{B} = \vec{V}B^2$$
$$\vec{V} = \frac{\vec{E} \times \vec{B}}{B^2} \quad \text{——— (10)}$$

This is our general formula.

As we know the Larmor radius is

$$r_L = \frac{V_\perp}{\omega_c}$$

The guiding centre drift caused by F can be written as,

By xing & ÷ing by q to eq. (10),

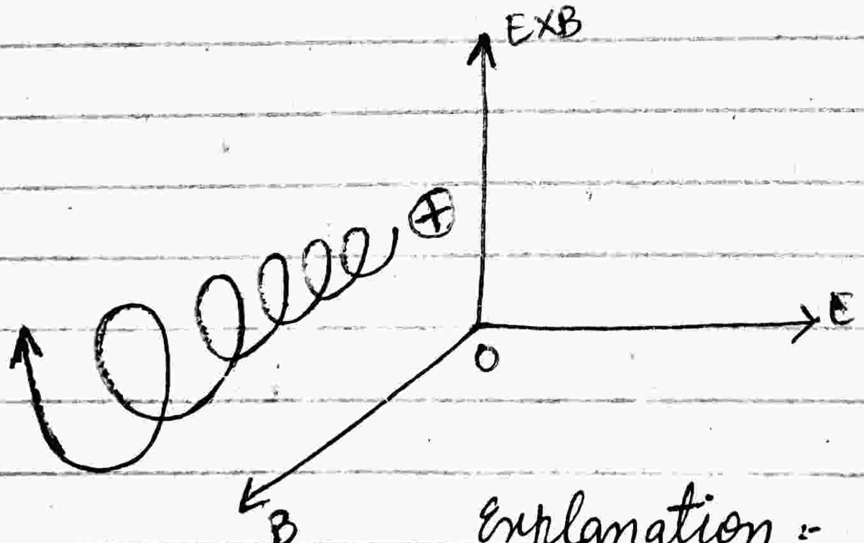
$$\vec{V} = \frac{\vec{E} \times \vec{B}}{B^2} \quad \text{—— (10)}$$

$$V_{fD} = \frac{q\vec{E} \times \vec{B}}{qB^2}$$

Now we replace qE by F as,

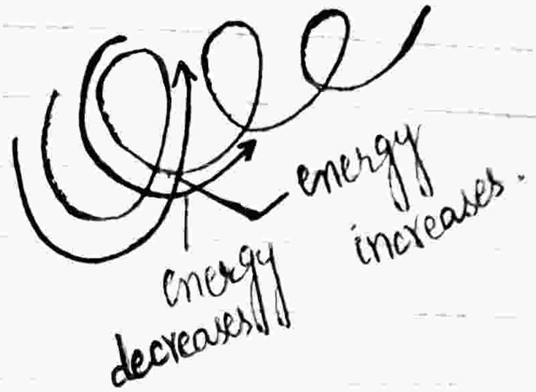
$\because F = qE$

$$V_{fD} = \frac{F \times B}{qB^2} \quad \text{—— (11)}$$



Explanation :-

During 1st half cycle, particle take energy from electric field. In other words we say $V_\perp$ is increasing it means Larmour radius also increasing.



∵ As radius decreases, velocity & the energy also decreases.

In the next half cycle particles lose energy it means $V_\perp$ decreases so, Larmour radius also decreases. This energy difference causes the drift velocity or the particle to move.

Now,

if we want to calculate the gravitational force drift then replace F by mg in eq. (11).

$$V_g = \frac{mg \times B}{qB^2}$$

For particle ($\bar{e}$) the above eq. can be written as,

$$V_{g_e} = \frac{-mg \times B}{qB^2} \quad \text{— (12)}$$

∵ m = electron
∵ M = Ion

For ion,

$$V_{g_i} = \frac{Mg \times B}{qB^2} \quad \text{— (13)}$$

Under gravitational force particle drift in opposite direction.

Current Density :-

As the particles drift in opposite direction so, the net current density in plasma is given by,

$$J = nqV \quad \text{(general)}$$

The net current density is

$$J = J_e + J_i$$

$$J = -nqV_{g_e} + nqV_{g_i}$$

Now put the value of V_{g_e} & V_{g_i} from eq. (12) & (13) ;

$$J = -nq\left(\frac{-m\vec{g}\times B}{qB^2}\right) + nq\left(\frac{M\vec{g}\times B}{qB^2}\right)$$

$$J = n\left(\frac{g\times B}{B^2}\right)\left[m + M\right]$$

By Rearrging,

$$J = n(M+m)\,\frac{g\times B}{B^2}$$

This is the final expression of current density. In this case, when the lines of force are curved, there is an effective gravitational field due to centrifugal force. It is independent of mass. The centrifugal force is the basis of plasma instability called the gravitational instability which has nothing to do with real gravity.