Electromagnetic Waves ; $B_0 = 0$, $B_1 \neq 0$.

In case of wave with $B_1 \neq 0$, these are transverse electromagnetic waves – light waves or radio waves travelling through a plasma. We begin with a brief review of light waves in a vacuum.

By applying Maxwell's eq.,

$$\nabla \times B_1 = \frac{4\pi}{c^2} J_1 + \frac{1}{c^2} \frac{\partial E_1}{\partial t} \qquad \text{—— (1)}$$

For electromagnetic waves, in Maxwell's eqs. both electric & magnetic fields are perturbed.

By another Maxwell's eq.,

$$\nabla \times E_1 = -\dot{B}_1 \quad \text{——— (2)}$$

differentiate eq. (1) w.r.t time,

$$\nabla \times \dot{B}_1 = \frac{4\pi}{c^2}\dot{J}_1 + \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2} \quad \text{——— (3)}$$

putting eq. (2) in eq. (3),

$$-[\nabla \times (\nabla \times E_1)] = \frac{4\pi}{c^2}\dot{J}_1 + \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2}$$

Then, according to identity of vector triple product,

$$a \times (b \times c) = ab^2 - b(ab)$$

$$-[\nabla(\nabla \cdot E_1) - \nabla^2 E_1] = \frac{4\pi}{c^2}\dot{J}_1 + \frac{1}{c^2}\frac{\partial^2 E_1}{\partial t^2}$$

Now let's applying sinsoidal character,

$$-[\partial/\partial x(\partial/\partial x \cdot E_1 e^{i(kx-\omega t)} - \partial^2/\partial x^2 E_1 e^{i(kx-\omega t)}] = \frac{4\pi}{c^2}\dot{J}_1 e^{i(kx-\omega t)} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E_1 e^{i(kx-\omega t)}$$

$$-[ik(ik \cdot E_1 e^{i(kx-\omega t)} - E_1(i^2k^2)e^{i(kx-\omega t)}] = \frac{4\pi}{c^2}\frac{\partial}{\partial t}J_1 e^{i(kx-\omega t)} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E_1 e^{i(kx-\omega t)}$$

$$-[ik(ik \cdot E_1) + E_1 k^2]e^{i(kx-\omega t)} = [\frac{4\pi}{c^2}J_1(-i\omega) + \frac{1}{c^2}(i^2\omega^2 E_1)]e^{i(kx-\omega t)}$$

$$-[ik(ik \cdot E_1) + k^2 E_1] = -\frac{4\pi}{c^2}i\omega J_1 + \frac{1}{c^2}(-\omega^2 E_1)$$

For the transverse wave $ik(ik \cdot E_1) = 0$ because they are parallel to each other. So, we are left with,

$$-c^2k^2E_1 = -4\pi i\omega J_1 - \omega^2 E_1$$

$$-c^2k^2E_1 + \omega^2 E_1 = -4\pi i\omega J_1$$

$$E_1(\omega^2 - c^2k^2) = -4\pi i\omega J_1 \quad \text{——— (4)}$$

For an electromagnetic waves propagating through a vacuum then, $J_1 = 0$ so, we can write now,

$$E_1(\omega^2 - c^2k^2) = 0$$

$$E_1 \neq 0$$

So,

$$\omega^2 - c^2k^2 = 0$$

$$\omega^2 = c^2k^2 .$$

This is the dispersion relation of light waves or photons through the vacuum.

It means when photons travel through a vacuum it only acts like a wave, not as a particle.

As ions don't take part in oscillations because of their heavy mass so,

Current Density $= J_1 = neV = n_0e(V_{i_1} - V_{e_1}) = -n_0eV_{e1}$ —— (5)

Let's write eq. of motion for $\bar{e}s$, (linearized).

$$mn_0 \frac{\partial V_{e1}}{\partial t} = -n_0eE_1$$

applying character,

$$mn_0 \frac{\partial}{\partial t} V_{e_1} e^{i(kx-\omega t)} = -n_0eE_1e^{i(kx-\omega t)}$$

$$m\cancel{n_0}(-i\omega)V_{e1}\cancel{e^{i(kx-\omega t)}} = -\cancel{n_0}eE_1\cancel{e^{i(kx-\omega t)}}$$

$$V_{e1} = \frac{eE_1}{mi\omega} = \frac{-ieE_1}{m\omega}$$

putting this value in eq. (5),

$$J_1 = -n_0eV_{e1}$$

$$J_1 = -n_0e\left(\frac{-ieE_1}{m\omega}\right)$$

$$J_1 = \frac{n_0e^2iE_1}{m\omega}$$

put this value in eq. (4),

∵ ions are low frequency waves bcoz its mass is high.
∵ $\bar{e}s$ are high frequency waves bcoz its mass is less.

$$E_1(\omega^2 - c^2K^2) = -4\pi i\omega\left(\frac{n_0 e^2 i E_1}{m\omega}\right)$$

$$\omega^2 - c^2K^2 = \frac{4\pi n_0 e^2}{m}$$

$$\omega^2 - c^2K^2 = \omega_{pe}^2$$

$$\omega^2 = \omega_{pe}^2 + c^2K^2 \qquad \text{--- (6)}$$

This is the dispersion relation of the electromagnetic wave or transverse wave in the plasma medium. So, it means photon or light wave has dual nature in plasma medium i.e., EMW or photons acts like both, as particle as well as a wave in a plasma medium. This dispersion relation like eq.(6) exhibits a phenomenon called cutoff.

- Electromagnetic waves are the only wave whose velocity is greater than velocity of light.

Dividing eq.(6) by K^2,

$$\frac{\omega^2}{K^2} = \frac{\omega_{pe}^2}{K^2} + c^2$$

phase velocity of EMW is more than the velocity of light.

Now, group velocity:

differentiate eq. (6) w.r.t K & ω,

$$\frac{d\omega}{dk} = ?$$

$$2\omega d\omega = 2Kc^2 dk$$

$$\frac{d\omega}{dK} = \frac{c^2K}{\omega}$$

$$\frac{d\omega}{dK} = \frac{c^2}{\omega/K}$$

As we know, $\omega/K = V_\phi$ So,

∵ Skin depth means how much wave will penetrate in plasma material. or

Skin depth is the measure of the penetration of a plane electromagnetic wave into a material.

$$\frac{d\omega}{dk} = \frac{c^2}{V_\varphi}$$

So, it means group velocity of EMW is less than phase velocity as well as velocity of light.

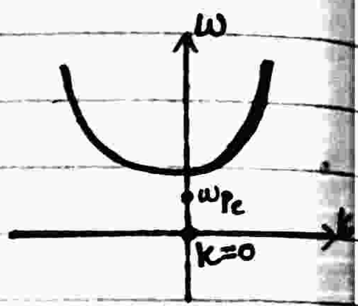
Dispersion relation of $\bar{e}$ plasma wave is:

$$\omega^2 = \omega_{Pe}^2 + \frac{3}{2} k^2 V_{th}^2 \quad \text{——— (7)}$$

By comparing eq. (6) & (7), we see that both dispersion relations are similar, only difference is that in eq. (6) there is velocity of light 'c' & in eq. (7) there is thermal velocity V_{th} & also $V_{th} << c$. Now from eq. (6) we have,

$$k^2 = \frac{\omega^2 - \omega_{Pe}^2}{c^2} \quad \text{——— (8)}$$

To find skin depth, we have three cases:



1- $\omega > \omega_{Pe}$ then k is +ive.
2- $\omega = \omega_{Pe}$ then $k = 0$
3- $\omega < \omega_{Pe}$ then k is imaginary.

Now, from here comes the concept of skin depth.

Eq. (8) can be written as:

$$K^2 = \frac{-(\omega_{Pe}^2 - \omega^2)}{c^2}$$

∵ $i^2 = -1$

$$K = \sqrt{\frac{-(\omega_{Pe}^2 - \omega^2)}{c^2}}$$

∵ $k = \frac{2\pi}{\lambda}$

$$k = i\frac{\sqrt{\omega_{Pe}^2 - \omega^2}}{c}$$

$$\text{Skin depth} = \frac{2\pi}{\lambda} = i\frac{\sqrt{\omega_{Pe}^2 - \omega^2}}{c}$$

$$\text{Skin depth} = \lambda_s = \frac{c}{2\pi\sqrt{\omega_{Pe}^2 - \omega^2}}$$

$$\text{Img } k = \text{Skin depth} = \frac{2\pi}{\lambda_s}$$

where, λ_s is the skin length.

as, $\omega_{Pe} = 4\pi n_0 e^2/m$

∵ n_0 is the variable So, if n_0 increases ω_{Pe} also increases.

∵ If ω is greater than ω_{Pe}, wave should propagate. ∵ If $\omega = \omega_{Pe}$, wave should not propagate.

∵ If ω is less than ω_{Pe}, then k is -ive or img.