Debye Shielding :-

A fundamental characteristics of the behaviour of the plasma is its ability to shield out electric potentials that are applied to it. Suppose we tried to put an electric field inside a plasma by inserting 2 charged balls connected to a battery (fig 1). The balls would attract particles of the opposite charge & almost immediately a cloud of ions would surround the -ive ball & a cloud of $\bar{e}$s would surround the +ive ball.

If the temperature is high, -ive & +ive charges comes out of sphere & makes field again, it means shielding is not perfect.

If the plasma were cold & there were no thermal motions, there would be just as many charges in the cloud as in the ball, the shielding would be perfect.

Let us compute the approximate thickness of such a charge cloud,

Poisson's Eq. is given by,

$$\nabla \cdot E = 4\pi\rho$$

ρ = charge density = ne

$n = \frac{N}{V}$ = no. density

Now,

write eq. for $\bar{e}$ & protons both,

$$\nabla_x \cdot E_x = 4\pi e n_i - 4\pi e n_e$$
$$\nabla_x \cdot E_x = 4\pi e (n_i - n_e)$$

As we know $E_x = -\nabla_x \phi$
put values

$$-\nabla_x^2 \phi = 4\pi e (n_i - n_e)$$
$$\nabla_x^2 \phi = -4\pi e (n_i - n_e)$$

$$\nabla_x^2 \phi = 4\pi e (n_e - n_i) \quad \text{—— (B)}$$

This is also the another form of Poisson's Eq.

From Maxwell's distribution fn.,

$$f(u) = A \exp\left(-mu^2/2KT\right)$$

When we have both thermal & electric field then we write,

$$f(u) = A \exp - \left(\frac{mu^2}{2KT} + \frac{q\phi}{KT}\right) \leftarrow P.E$$

This is Maxwell's Boltzmann's distribution fn.

∵ When potential multiply by charge it becomes energy.

∵ Electric field has P.E because it move particle from one place to another.

Now,
we assume that ions are so far from $\bar{e}$s

that they have infinite distance b/w them,
so,

$$n_i = n_\infty = n_0$$

∵ n_∞ means ions are at infinite distance from $\bar{e}s$.

$$\nabla_x^2 \varphi = 4\pi e(n_e - n_\infty) \quad \text{—— (1)}$$

Now,

we find no. density of $\bar{e}s$,

$$n_e = n_\infty \exp\left(\frac{e\varphi}{KT_e}\right)$$

put in eq. (1).

$$\nabla_x^2 \varphi = 4\pi e\, n_\infty \left(\exp \frac{e\varphi}{KT_e} - 1\right)$$

Applying series expansion, on above eq.

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots$$

By ignoring higher terms, we get,

$$\nabla_x^2 \varphi = 4\pi e n_\infty \left(\cancel{1} + \frac{e\varphi}{KT_e} - \cancel{1}\right)$$

$$\nabla_x^2 \varphi = \frac{4\pi e^2 n_\infty \varphi}{KT_e}$$

$$\frac{d^2\varphi}{dx^2} = \frac{4\pi e^2 n_\infty \varphi}{KT_e} \quad \text{—— (2)}$$

Dimension of eq. ② is,

$$\frac{4\pi e^2 n_\infty}{KTe} = \frac{1}{L^2}$$

or in this case,

$$\frac{4\pi e^2 n_\infty}{KTe} = \frac{1}{\lambda_D^2} \quad \text{——— (A)}$$

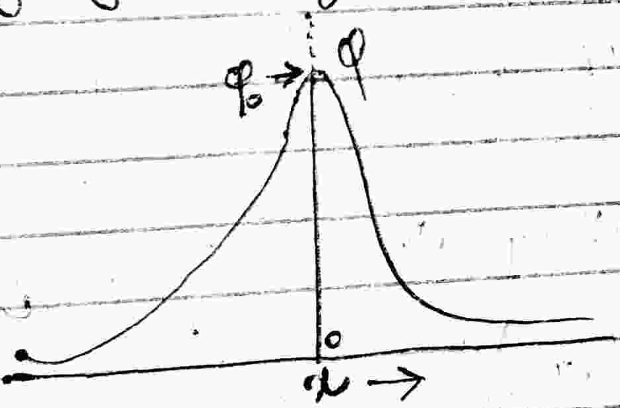
putting values in eq. ②, we get

$$\frac{d^2\varphi}{dx^2} = \frac{1}{\lambda_D^2}\varphi$$

The soln. of the above eq. is,

$$\varphi = \varphi_0 \exp\left(\frac{-x}{\lambda_D}\right) \quad \text{——— ③}$$

plotting graph of eq. ③,



$x = 0$
then, $\varphi = \varphi_0$

or when $x = 0$
then $e^0 = 1$
we have max. potential.

Now,

From eq. (A),

$$\lambda_D^2 = \frac{KT_e}{4\pi e^2 n_\infty} \quad \text{--- (4)}$$

For a gas, to be plasma : 1^{st} condition

$$\lambda_D <<< L$$

In this situation,

$\varphi = 0$ in poisson's eq. (B), so

$$4\pi e (n_e - n_i) = 0$$

$$n_e - n_i = 0$$

because $4\pi e$ is constant, it can't be zero.

$$n_e = n_i$$

This is the charge neutrality condition.

Taking eq. (4),

$$\lambda_D^2 = \frac{KT_e}{4\pi e^2 n_\infty}$$

When temperature is calculated in degrees then,

$$\lambda_D = \left(\frac{K}{4\pi e^2}\right)^{1/2} \left(\frac{T_e}{n_0}\right)^{1/2} \qquad \because n_i = n_\infty = n_0$$

$$\lambda_D = 69 \left(\frac{T_e}{n_0}\right)^{1/2} m \qquad (T \text{ in } ^\circ C)$$

In energy unit,

$$\lambda_D = \left(\frac{1}{4\pi e^2}\right)^{1/2} \left(\frac{KT_e}{n_0}\right)^{1/2}$$

$$\lambda_D = 7430\left(\frac{KT_e}{n_o}\right)^{1/2} \quad (T \text{ in eV})$$

2nd condition: For a gas to be in plasma,

$$n = \frac{N_D}{V} \Rightarrow N_D = nV$$

$\because$ vol of sphere $= \frac{4}{3}\pi r^3$
$\because$ Radius of sphere $= \lambda_D$
$r = \lambda_D$

So,

$$N_D = n\frac{4}{3}\pi\lambda_D^3$$

putting values of λ_D, we get

$$N_D = n\frac{4\pi}{3}\left(\frac{KT_e}{4\pi n_o e^2}\right)^{3/2}$$

$$= \frac{1}{3}(4\pi)^{1-3/2}\, n^{1-3/2}\left(\frac{KT_e}{e^2}\right)^{3/2}$$

$$= \frac{1}{3}\frac{1}{(4\pi)^{1/2}}\frac{1}{n^{1/2}}\left(\frac{KT_e}{e^2}\right)^{3/2}$$

$$= \frac{1}{3}\frac{1}{(4\pi)^{1/2}}\frac{K^{3/2}}{e^3}\frac{(T_e)^{3/2}}{(n)^{1/2}}$$

$$N_D = 1.38\times10^6\frac{(T_e)^{3/2}}{(n)^{1/2}} \quad (\text{temp in } ^\circ C)$$

It means in debye's sphere the no. of particles should be much greater than 1

$$N_D >>> 1$$

This is the second condition.

3rd Condition :-

τ is the collision time of the plasma particles with neutral particles.

$$\omega\tau > 1$$
$$\omega\tau > 1$$

So,

$$\omega > \frac{1}{\tau}$$
$$\omega > \tau^{-1}$$

Hence,

there are 3 conditions for a gas to be plasma, or these are the 3 conditions, a plasma must satisfy :

1- $\lambda_D \ll L$

2- $N_D \gg 1$

3- $\omega\tau > 1$

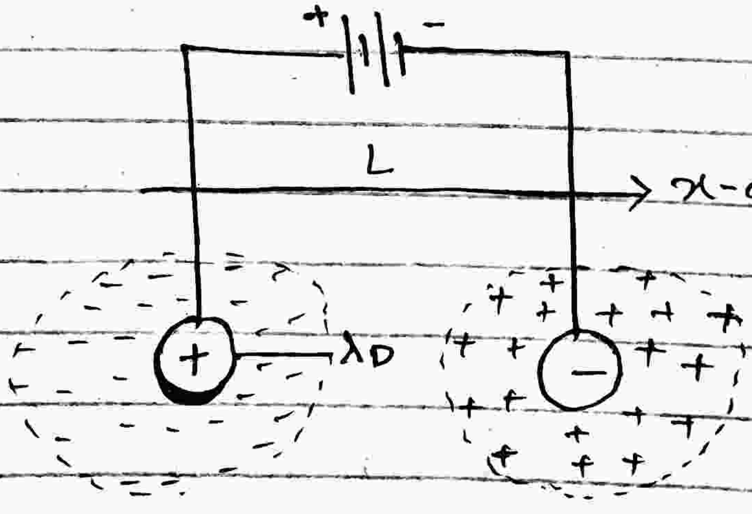


figure 1