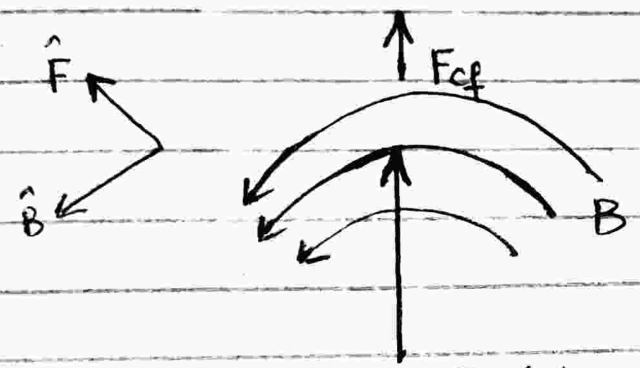
Curved B or Curvature Drift :-

We assume that the lines of force are curved with a constant radius of curvature R_c & take $|B|$ to be constant. Such a field does not obey Maxwell's eq. in vacuum. A guiding centre drift arises from the centrifugal force feel by the particles.

Figure :-



A curved magnetic Field.

Here, we use cylinderical coordinates (r, θ, z) because we have curved B.

The general formula of centrifugal force is,

$$F = \frac{-mv^2}{r}$$

If $V_{\parallel}^2$ denotes the average square of the component of random velocity along B, the average centrifugal force will be

$$F_{cf} = \frac{-mv_{\parallel}^2}{R_c} \hat{r} \qquad \text{(1)}$$

where, $\vec{R_c} = |R_c| \hat{r}$

or,

$$\hat{r} = \frac{\vec{R_c}}{|R_c|}$$

put in eq. ①,

$$F_{cf} = -\frac{mv_{\parallel}^2}{R_c}\frac{\vec{R_c}}{R_c}$$

$$F_{cf} = -\frac{mv_{\parallel}^2 \vec{R_c}}{R_c^2} \quad \text{— ②}$$

Curvature drift is defined as, from general formula,

$$V = \frac{E \times B}{B^2}$$

xing & ÷ing by q,

$$V = \frac{qE \times B}{qB^2}$$

$$V_R = \frac{F_{cf} \times B}{qB^2} \quad \text{— ③}$$

put the value from eq.(2) in eq.(3).

$$V_R = \mp \frac{mv_{\parallel}^2}{qB^2}\frac{\vec{R_c} \times \vec{B}}{R_c^2} \quad \text{— ④}$$

when the magnetic field lines are curved then, by using Maxwell's eq. in vacuum,

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c^2} J + \frac{1}{c}\frac{\partial E}{\partial t} \quad \text{— ⑤}$$

where J is the current density.
As we are concerned with the magnetic field so.
eq. (5) becomes, also we don't have any electric field variation.

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c^2} J \quad \text{—— (6)}$$

as, $J = nqV$

here, $n = 0$

because this particle is in vacuum so, $J = 0$ for vaccum. (no particle present).

So, eq. (6) reduces to,

$$\vec{\nabla} \times \vec{B} = 0$$

Since, we are concerned with curved coordinates so, we assume that, B_θ exist.

then B_r & $B_z = 0$

As, 'B' has only a θ- component & ∇B has only r- component then we have,

$$(\nabla \times B)_z = \frac{1}{r}\left(\frac{\partial}{\partial r} rB_\theta\right) = 0 \quad \text{—— (7)}$$

$\frac{1}{r}$ can't be zero because it is the radius of curvature.

So, $$\frac{\partial}{\partial r} rB_\theta = 0$$

$$rB_\theta = C \quad \text{(constant)}.$$

Now, Integrate eq. (7),

$$\int \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta)\, d\theta = 0\int d\theta$$

$$r B_\theta = C$$ → which is the constant of integration.

$$B_\theta = \frac{C}{r}$$

$\because r = R_c$

$$B_\theta \propto \frac{1}{R_c}$$

$$\nabla B = \frac{1}{R_c^2}$$

$$\frac{\nabla B}{B} = \frac{\vec{R_c}}{R_c^2}$$

$$\frac{\nabla B}{B} = \frac{\frac{1}{R_c^2}}{\frac{1}{\vec{R_c}}} = \frac{\vec{R_c}}{R_c^2}$$

$$\nabla B = \frac{B \vec{R_c}}{R_c^2} \quad \text{——— (8)}$$

$$\because V_{\nabla B} = \pm \frac{r_L V_\perp}{2} \left(\frac{\vec{B \times \nabla B}}{B^2} \right)$$

put value from eq. (8) in above eq.

$$V_{\nabla B} = \pm \frac{r_L V_\perp}{2} \frac{B \times |B| R_c}{R_c^2 B^2}$$

$$\because r_L = \frac{V_\perp}{\omega_c}$$

$$V_{\nabla B} = \pm \frac{r_L V_\perp}{2} \frac{B \times R_c \; |B|}{R_c^2 \times B^{2}}$$

$$V_{\nabla B} = \mp \frac{1}{2} \frac{V_\perp^2}{\omega_c B} \frac{R_c \times B}{R_c^2} \quad \text{——— (9)}$$

$\because \omega_c = qB/m$ (cyclotron frequency.)

So, eq. (9) becomes,

$$V_{\nabla B} = \mp \frac{1}{2} \frac{V_\perp^2}{qB^2/m} \frac{R_c \times B}{R_c^2}$$

$$V_{\nabla B} = \mp \frac{1}{2} \frac{mV_\perp^2}{qB^2} \frac{R_c \times B}{R_c^2} \quad \text{—— (10)}$$

when magnetic field lines are curved then two types of drift arises,

By adding eq. (4) & (10) we get,

Total drift which can be written as,

$$V_R + V_{\nabla B} = \frac{m}{qB^2} \frac{R_c \times B}{R_c^2} \left(V_\parallel^2 + \frac{1}{2} V_\perp^2 \right) \quad \text{—— (11).}$$

This is the required result.