3.7 Laminar and Turbulent Flows

It is obvious looking at the friction factor diagram (Figure 3.10) that something happens to the flow when the Reynolds number exceeds about 2000. It turns out that the frictional losses in a fluid flow depend on whether the flow is laminar or turbulent. In order to understand the difference between these flows, let us review an experiment first conducted by Sir Osborne Reynolds (1843–1912). Reynolds was an English civil engineer noted for both his theoretical and applied studies in fluid mechanics. Reynolds's experiment used a simple apparatus to visualize a flow field (Figure 3.11). In this apparatus, dye is injected along the centerline of a pipe in which fluid is flowing. The dye serves to show the nature of the flow as the fluid moves along the tube. At low velocities the dye remains approximately on the centerline, traveling downstream with, and at the velocity of, the flow. Because of molecular diffusion there will be a small flux of dye away from the centerline, but this will be so small that no departure from the line will be observed with the naked eye. Reynolds characterized this flow situation by describing the fluid as moving in distinct *laminae* (Latin for layers), and today we refer to such flow as **laminar flow**.

Now consider what happens as the velocity of fluid flow in the tube is increased. At some value of increased velocity, the dye is observed to become mixed within small, randomly located spots within the flow; with higher velocities the dye rapidly becomes completely mixed with the surrounding fluid in a short distance of flow. Reynolds referred to this flow as sinuous or disturbed flow. Today we refer to flows exhibiting such violent mixing as **turbulent flows**. In this case the mixing takes place much more rapidly than realized by molecular diffusion, and in fact we call this mixing *turbulent diffusion*. We can conceptualize the turbulent mixing as small parcels of the fluid "jumping" away from a streamline to a different portion of the flow field. Most flows we observe in nature, for example, clouds, rivers, ocean waves, are turbulent flows. In this regard, we note that "laminar" and "turbulent" refer to properties of the flow, and not properties of the fluid. For example, the water in the garden hose could be in either laminar or turbulent flow without changing its properties, that is, density or viscosity.

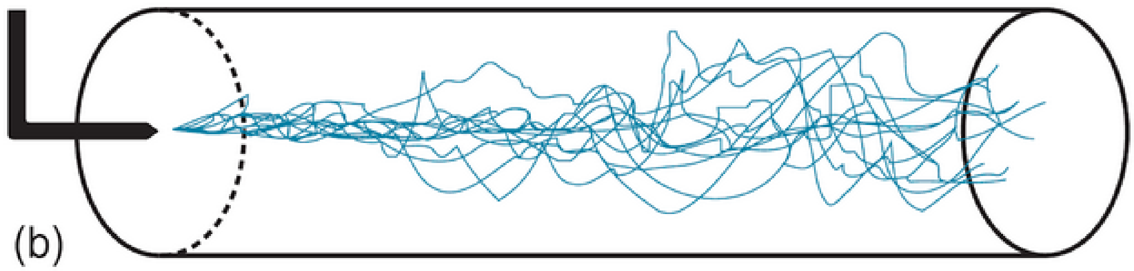

Figure 3.11 Reynolds's experiment. In laminar flow (*a*), the dye remains a thin line, moving with the fluid. The dye trace becomes convoluted in turbulent flow (*b*).

To explain the last statement, we return to Reynolds's experiment. By carefully changing the properties of the fluids in his apparatus, as well as the tube diameter and flow velocity, Reynolds empirically defined a dimensionless number **R** which describes the flow properties:

$$\mathbf{R} = \frac{UL\rho}{\mu} \qquad \textbf{(3.34)}$$

where U = a characteristic velocity [L T^{-1}]; L = a characteristic length [L]; ρ = fluid density [M L^{-3}]; μ = viscosity [M L^{-1} T^{-1}]. **R** is called the **Reynolds number**. According to Figure 3.10, if **R** is small (with U taken as the average velocity and L taken as the diameter of the tube), say less than 2000, then the flow in a pipe will be laminar. If **R** is large, say greater than 4000, then the flow will be turbulent. At values between these limits, called the transition region, the flow is not easily characterized as one or the other. The choice of the velocity and length is a matter of accepted convention. For circular pipes such as the garden hose, the characteristic velocity is taken as the mean value over the cross section (the discharge divided by the cross-sectional area), and the characteristic length as the hose diameter. If water flows at 15°C at 2 m s^{-1}, through a 30-mm diameter hose, the Reynolds number would be calculated as follows:

$U = 2\ \text{m s}^{-1}$ (mean velocity);

$L = 0.03\ \text{m}$ (pipe diameter);

$\rho = 10^3\ \text{kg m}^{-3}$;

$\mu = 1.139 \times 10^{-3}\ \text{Pa} \cdot \text{s}$;

$$\mathbf{R} = \frac{(2)(0.03)(10^3)}{(1.139 \times 10^{-3})} \approx 53{,}000,$$

which indicates that the flow is turbulent. The flow could be made laminar by somehow reducing **R** to some value less than 2000. We could accomplish this by:

a. reducing the velocity,
b. reducing the diameter,
c. reducing the density of the fluid, or
d. increasing the viscosity.

In order to accomplish (c) or (d), we would have to change the fluid temperature, or perhaps the fluid itself. This latter alternative suggests another important conclusion from Reynolds's experiments. Two *different* fluids with the same Reynolds number will have similar flows, that is, laminar or turbulent. Thus, we can expect the same degree of mixing or turbulence inside the hose for both water and glycerin, if they both have the same value for **R**. Because the viscosity of glycerin is much greater than that of water, it would be difficult to achieve this similarity in practice.

Now that we have some feeling for the meaning of the Reynolds number, let us reconsider the differences between laminar and turbulent flow as evidenced by Reynolds's experiment. For the laminar flow, any time the dye began to stray from the centerline of the tube, it was hindered, or its motion was restored to the centerline. The attempted deviation or acceleration is an apparent inertial force, and the restoring force is due to the viscous forces within the fluid. Thus, for laminar flow, the mixing is restrained as the viscous force must dominate over the inertial force. If the inertial force becomes greater than the viscous force, then the dye would have a tendency to deviate from the centerline, and the flow could become turbulent. We can therefore take the ratio of the inertial to viscous force as an index of when flow would be expected to be laminar or turbulent. If the viscous forces >> inertial forces, the flow is laminar, and if the inertial forces >> viscous forces, the flow is turbulent. We know that the inertial force is proportional to mass × acceleration, which could be expressed as $(\rho)(L^2)(\text{velocity}^2)$. Similarly, the viscous force is proportional to the shear stress × area, which could be expressed as $(\mu)(L)(\text{velocity})$. Thus,

$$\frac{\text{inertial force}}{\text{viscous force}} \propto \frac{\rho L^2 V^2}{\mu L V} = \frac{\rho L V}{\mu} = \mathbf{R}. \qquad \textbf{(3.35)}$$

The Reynolds number is equal to the ratio of the inertial to viscous forces within the fluid.

Summary

- The Reynolds number $\mathbf{R} = \rho U D/\mu$ is a measure of the relative importance of viscous forces in a flow. When the Reynolds number is less than 2000 for pipe flow, viscous forces are large enough to damp any disturbance in the flow resulting in laminar flow. Viscous forces are less effective at higher Reynolds numbers (above about 4000 for pipes) so that disturbances to the flow can grow, causing the flow to become turbulent. Two different fluids with the same Reynolds number will have similar flows. {Section 3.7}