Subject: Signals and Systems

Chapter: 09

The Laplace Transform

Text Book: Signals & Systems By: Alan V. Oppenheim, Alan S. Willsky with S. Hamid Nawab, 2nd Edition

9.1 THE LAPLACE TRANSFORM

The response of a linear time-invariant system with impulse response $h(t)$ to a complex exponential input of the form e^{st} is

$$y(t) = H(s)e^{st}, \tag{9.1}$$

where

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt. \tag{9.2}$$

For s imaginary (i.e., $s = j\omega$), the integral in eq. (9.2) corresponds to the Fourier transform of $h(t)$. For general values of the complex variable s, it is referred to as the *Laplace transform* of the impulse response $h(t)$.

The Laplace transform of a general signal $x(t)$ is defined as[1]

$$\boxed{X(s) \triangleq \int_{-\infty}^{+\infty} x(t)e^{-st}\,dt,} \tag{9.3}$$

and we note in particular that it is a function of the independent variable s corresponding to the complex variable in the exponent of e^{-st}. The complex variable s can be written as $s = \sigma + j\omega$, with σ and ω the real and imaginary parts, respectively. For convenience, we will sometimes denote the Laplace transform in operator form as $\mathfrak{L}\{x(t)\}$ and denote the transform relationship between $x(t)$ and $X(s)$ as

$$x(t) \overset{\mathfrak{L}}{\longleftrightarrow} X(s). \tag{9.4}$$

Cont.

When $s = j\omega$, eq. (9.3) becomes

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt, \tag{9.5}$$

which corresponds to the *Fourier transform* of $x(t)$; that is,

$$X(s)\big|_{s=j\omega} = \mathfrak{F}\{x(t)\}. \tag{9.6}$$

The Laplace transform also bears a straightforward relationship to the Fourier transform when the complex variable s is not purely imaginary. To see this relationship, consider $X(s)$ as specified in eq. (9.3) with s expressed as $s = \sigma + j\omega$, so that

$$X(\sigma + j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-(\sigma + j\omega)t}\,dt, \tag{9.7}$$

or

$$X(\sigma + j\omega) = \int_{-\infty}^{+\infty} [x(t)e^{-\sigma t}]e^{-j\omega t}\,dt. \tag{9.8}$$

We recognize the right-hand side of eq. (9.8) as the Fourier transform of $x(t)e^{-\sigma t}$; that is, the Laplace transform of $x(t)$ can be interpreted as the Fourier transform of $x(t)$ after multiplication by a real exponential signal. The real exponential $e^{-\sigma t}$ may be decaying or growing in time, depending on whether σ is positive or negative.

Example 9.1

Let the signal $x(t) = e^{-at}u(t)$. From Example 4.1, the Fourier transform $X(j\omega)$ converges for $a > 0$ and is given by

$$X(j\omega) = \int_{-\infty}^{+\infty} e^{-at}u(t)e^{-j\omega t}\,dt = \int_{0}^{\infty} e^{-at}e^{-j\omega t}\,dt = \frac{1}{j\omega + a}, \quad a > 0. \tag{9.9}$$

From eq. (9.3), the Laplace transform is

$$X(s) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt = \int_{0}^{\infty} e^{-(s+a)t}dt, \tag{9.10}$$

or, with $s = \sigma + j\omega$,

$$X(\sigma + j\omega) = \int_{0}^{\infty} e^{-(\sigma+a)t}e^{-j\omega t}\,dt. \tag{9.11}$$

By comparison with eq. (9.9) we recognize eq. (9.11) as the Fourier transform of $e^{-(\sigma+a)t}u(t)$, and thus,

$$X(\sigma + j\omega) = \frac{1}{(\sigma + a) + j\omega}, \quad \sigma + a > 0, \tag{9.12}$$

or equivalently, since $s = \sigma + j\omega$ and $\sigma = \Re e\{s\}$,

$$X(s) = \frac{1}{s + a}, \quad \Re e\{s\} > -a. \tag{9.13}$$

Cont.

That is,

$$e^{-at}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \quad \Re e\{s\} > -a. \tag{9.14}$$

For example, for $a = 0$, $x(t)$ is the unit step with Laplace transform $X(s) = 1/s$, $\Re e\{s\} > 0$.

We note, in particular, that just as the Fourier transform does not converge for all signals, the Laplace transform may converge for some values of $\Re e\{s\}$ and not for others. In eq. (9.13), the Laplace transform converges only for $\sigma = \Re e\{s\} > -a$. If a is positive, then $X(s)$ can be evaluated at $\sigma = 0$ to obtain

$$X(0 + j\omega) = \frac{1}{j\omega + a}. \tag{9.15}$$

As indicated in eq. (9.6), for $\sigma = 0$ the Laplace transform is equal to the Fourier transform, as is evident in the preceding example by comparing eqs. (9.9) and (9.15). If a is negative or zero, the Laplace transform still exists, but the Fourier transform does not.

Example 9.2

For comparison with Example 9.1, let us consider as a second example the signal

$$x(t) = -e^{-at}u(-t). \tag{9.16}$$

Then

$$\begin{aligned} X(s) &= -\int_{-\infty}^{\infty} e^{-at}e^{-st}u(-t)\,dt \\ &= -\int_{-\infty}^{0} e^{-(s+a)t}\,dt, \end{aligned} \tag{9.17}$$

or

$$X(s) = \frac{1}{s+a}. \tag{9.18}$$

For convergence in this example, we require that $\mathcal{Re}\{s+a\} < 0$, or $\mathcal{Re}\{s\} < -a$; that is,

$$-e^{-at}u(-t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+a}, \qquad \mathcal{Re}\{s\} < -a. \tag{9.19}$$

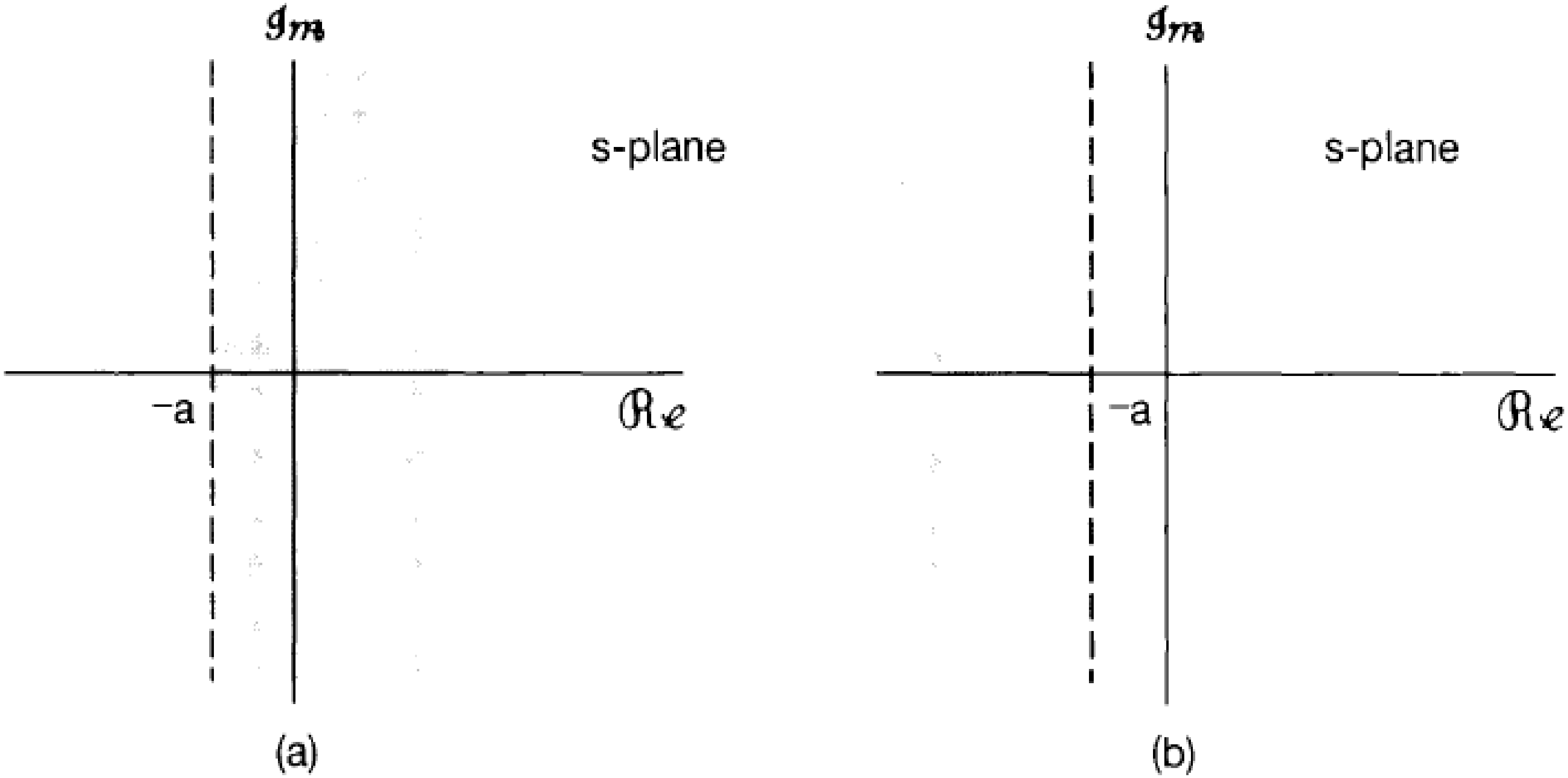

Figure 9.1 (a) ROC for Example 9.1; (b) ROC for Example 9.2.

Example 9.3

In this example, we consider a signal that is the sum of two real exponentials:

$$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t). \tag{9.20}$$

The algebraic expression for the Laplace transform is then

$$\begin{aligned} X(s) &= \int_{-\infty}^{\infty} \left[3e^{-2t}u(t) - 2e^{-t}u(t)\right] e^{-st}\, dt \\ &= 3\int_{-\infty}^{\infty} e^{-2t}e^{-st}u(t)\, dt - 2\int_{-\infty}^{\infty} e^{-t}e^{-st}u(t)\, dt. \end{aligned} \tag{9.21}$$

Each of the integrals in eq. (9.21) is of the same form as the integral in eq. (9.10), and consequently, we can use the result in Example 9.1 to obtain

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}. \tag{9.22}$$

Cont.

To determine the ROC we note that $x(t)$ is a sum of two real exponentials, and from eq. (9.21) we see that $X(s)$ is the sum of the Laplace transforms of each of the individual terms. The first term is the Laplace transform of $3e^{-2t}u(t)$ and the second term the Laplace transform of $-2e^{-t}u(t)$. From Example 9.1, we know that

$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+1}, \qquad \mathcal{Re}\{s\} > -1,$$

$$e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \qquad \mathcal{Re}\{s\} > -2.$$

The set of values of $\mathcal{Re}\{s\}$ for which the Laplace transforms of both terms converge is $\mathcal{Re}\{s\} > -1$, and thus, combining the two terms on the right-hand side of eq. (9.22), we obtain

$$3e^{-2t}u(t) - 2e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{s-1}{s^2+3s+2}, \quad \mathcal{Re}\{s\} > -1. \tag{9.23}$$

Example 9.4

In this example, we consider a signal that is the sum of a real and a complex exponential:

$$x(t) = e^{-2t}u(t) + e^{-t}(\cos 3t)u(t). \tag{9.24}$$

Using Euler's relation, we can write

$$x(t) = \left[e^{-2t} + \frac{1}{2}e^{-(1-3j)t} + \frac{1}{2}e^{-(1+3j)t}\right]u(t),$$

and the Laplace transform of $x(t)$ then can be expressed as

$$\begin{aligned} X(s) = & \int_{-\infty}^{\infty} e^{-2t}u(t)e^{-st}\,dt \\ & + \frac{1}{2}\int_{-\infty}^{\infty} e^{-(1-3j)t}u(t)e^{-st}\,dt \\ & + \frac{1}{2}\int_{-\infty}^{\infty} e^{-(1+3j)t}u(t)e^{-st}\,dt. \end{aligned} \tag{9.25}$$

Cont.

Each of the integrals in eq. (9.25) represents a Laplace transform of the type encountered in Example 9.1. It follows that

$$e^{-2t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}, \quad \mathcal{Re}\{s\} > -2, \tag{9.26}$$

$$e^{-(1-3j)t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+(1-3j)}, \quad \mathcal{Re}\{s\} > -1, \tag{9.27}$$

$$e^{-(1+3j)t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+(1+3j)}, \quad \mathcal{Re}\{s\} > -1. \tag{9.28}$$

For all three Laplace transforms to converge simultaneously, we must have $\mathcal{Re}\{s\} > -1$. Consequently, the Laplace transform of $x(t)$ is

$$\frac{1}{s+2} + \frac{1}{2}\left(\frac{1}{s+(1-3j)}\right) + \frac{1}{2}\left(\frac{1}{s+(1+3j)}\right), \quad \mathcal{Re}\{s\} > -1, \tag{9.29}$$

or, with terms combined over a common denominator,

$$e^{-2t}u(t) + e^{-t}(\cos 3t)u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}, \quad \mathcal{Re}\{s\} > -1. \tag{9.30}$$

Cont.

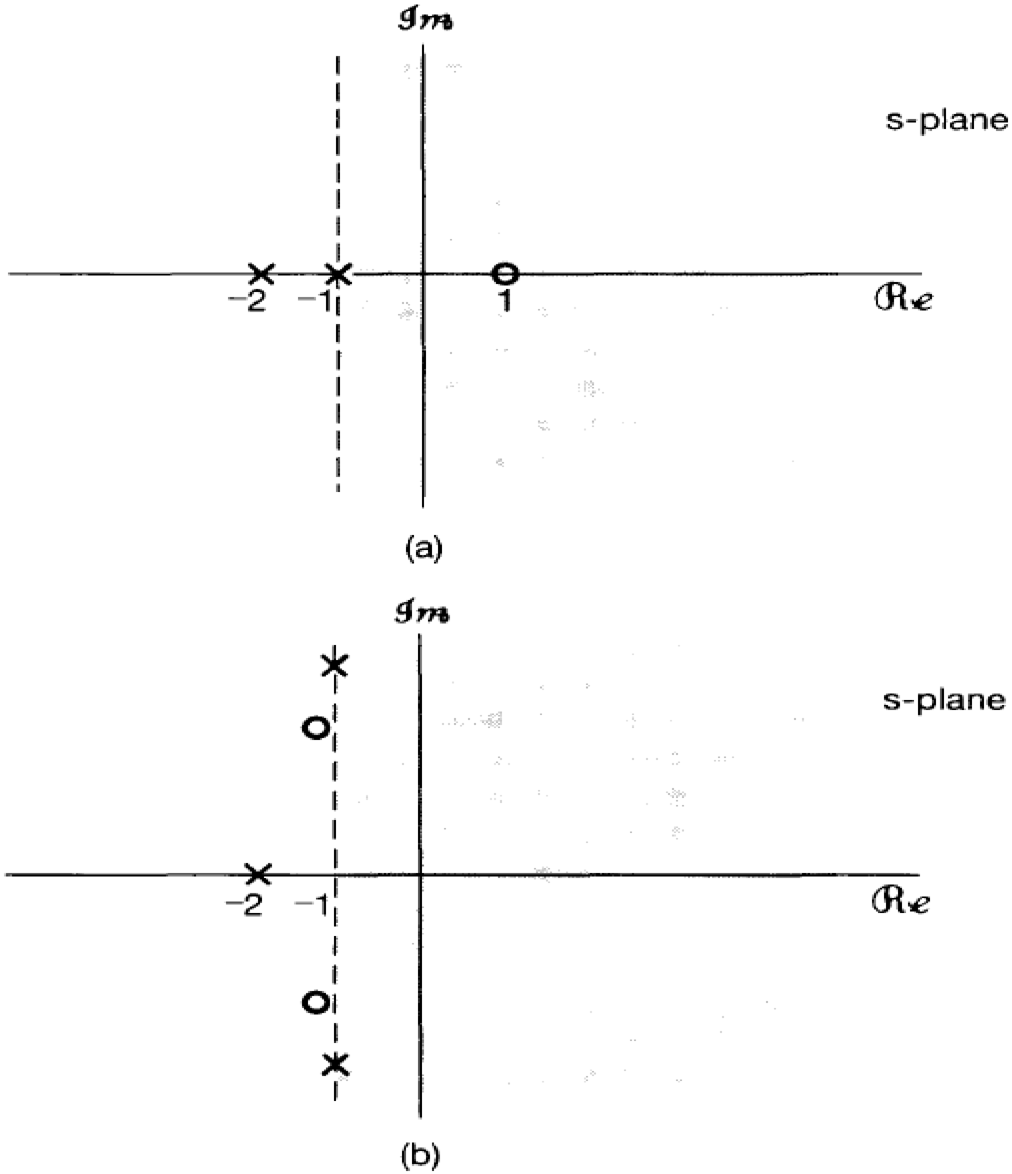

Figure 9.2 *s*-plane representation of the Laplace transforms for (a) Example 9.3 and (b) Example 9.4. Each × in these figures marks the location of a pole of the corresponding Laplace transform—i.e., a root of the denominator. Similarly, each ∘ marks a zero—i.e., a root of the the numerator. The shaded regions indicate the ROCs.

Example 9.5

Let

$$x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t). \tag{9.32}$$

The Laplace transform of the second and third terms on the right-hand side of eq. (9.32) can be evaluated from Example 9.1. The Laplace transform of the unit impulse can be evaluated directly as

$$\mathfrak{L}\{\delta(t)\} = \int_{-\infty}^{+\infty} \delta(t)e^{-st}\,dt = 1, \tag{9.33}$$

which is valid for any value of s. That is, the ROC of $\mathfrak{L}\{\delta(t)\}$ is the entire s-plane. Using this result, together with the Laplace transforms of the other two terms in eq. (9.32), we obtain

$$X(s) = 1 - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2}, \quad \Re e\{s\} > 2, \tag{9.34}$$

Cont.

$$X(s) = \frac{3\,(s+1)\,3\,(s-2) - 12\,(s-2) + 3\,(s+1)}{3\,(s+1)\,3\,(s-2)}$$

$$X(s) = \frac{3\,(s+1)\,3\,(s-2) - 12\,(s-2) + 3\,(s+1)}{9\,(s+1)\,(s-2)}$$

$$X(s) = \frac{9\,(s^2 - s - 2) - 12s + 24 + 3s + 3}{9\,(s+1)\,(s-2)}$$

$$X(s) = \frac{9s^2 - 9s - 18 - 12s + 24 + 3s + 3}{9\,(s+1)\,(s-2)}$$

$$X(s) = \frac{9s^2 - 18s + 9}{9\,(s+1)\,(s-2)}$$

$$X(s) = \frac{9\,(s^2 - 2s + 1)}{9\,(s+1)\,(s-2)}$$

$$X(s) = \frac{(s^2 - 2s + 1)}{(s+1)\,(s-2)}$$

$$X(s) = \frac{(s-1)^2}{(s+1)\,(s-2)}$$

Cont.

$$X(s) = \frac{(s-1)^2}{(s+1)(s-2)}, \quad \Re e\{s\} > 2, \tag{9.35}$$

where the ROC is the set of values of s for which the Laplace transforms of all three terms in $x(t)$ converge. The pole-zero plot for this example is shown in Figure 9.3, together with the ROC. Also, since the degrees of the numerator and denominator of $X(s)$ are equal, $X(s)$ has neither poles nor zeros at infinity.

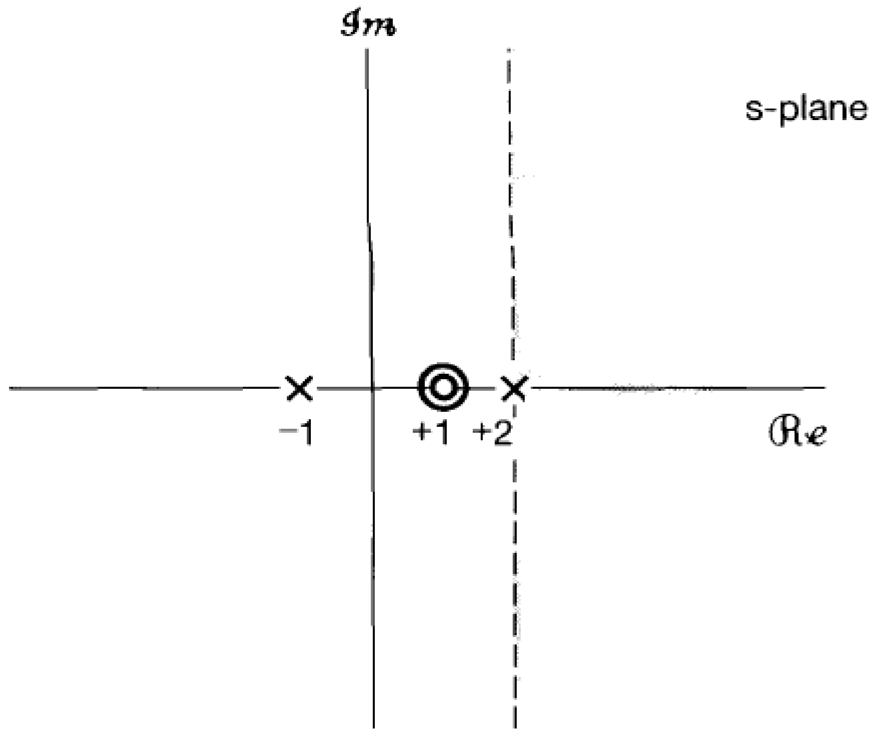

Figure 9.3 Pole-zero plot and ROC for Example 9.5.

Properties of the Laplace Transform

9.5.1 Linearity of the Laplace Transform

If

$$x_1(t) \stackrel{\mathcal{L}}{\longleftrightarrow} X_1(s) \quad \text{with a region of convergence that will be denoted as } R_1$$

and

$$x_2(t) \stackrel{\mathcal{L}}{\longleftrightarrow} X_2(s) \quad \text{with a region of convergence that will be denoted as } R_2,$$

then

$$ax_1(t) + bx_2(t) \stackrel{\mathcal{L}}{\longleftrightarrow} aX_1(s) + bX_2(s), \text{ with ROC containing } R_1 \cap R_2. \tag{9.82}$$

Cont.

9.5.2 Time Shifting

If

$$x(t) \stackrel{\mathcal{L}}{\longleftrightarrow} X(s), \qquad \text{with ROC} = R,$$

then

$$\boxed{x(t - t_0) \stackrel{\mathcal{L}}{\longleftrightarrow} e^{-st_0}X(s), \quad \text{with ROC} = R.} \tag{9.87}$$

Cont.

9.5.3 Shifting in the s-Domain

If

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s), \qquad \text{with ROC} = R,$$

then

$$\boxed{e^{s_0 t} x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s - s_0), \quad \text{with ROC} = R + \mathcal{R}e\{s_0\}.} \tag{9.88}$$

That is, the ROC associated with $X(s - s_0)$ is that of $X(s)$, shifted by $\mathcal{R}e\{s_0\}$. Thus, for any value s that is in R, the value $s + \mathcal{R}e\{s_0\}$ will be in R_1. This is illustrated in Figure 9.23. Note that if $X(s)$ has a pole or zero at $s = a$, then $X(s - s_0)$ has a pole or zero at $s - s_0 = a$—i.e., $s = a + s_0$.

An important special case of eq. (9.88) is when $s_0 = j\omega_0$—i.e., when a signal $x(t)$ is used to modulate a periodic complex exponential $e^{j\omega_0 t}$. In this case, eq. (9.88) becomes

$$e^{j\omega_0 t} x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s - j\omega_0), \quad \text{with ROC} = R. \tag{9.89}$$

The right-hand side of eq. (9.89) can be interpreted as a shift in the s-plane parallel to the $j\omega$-axis. That is, if the Laplace transform of $x(t)$ has a pole or zero at $s = a$, then the Laplace transform of $e^{j\omega_0 t} x(t)$ has a pole or zero at $s = a + j\omega_0$.

Cont.

9.5.4 Time Scaling

If

$$x(t) \overset{\mathfrak{L}}{\longleftrightarrow} X(s), \quad \text{with ROC} = R,$$

then

$$\boxed{x(at) \overset{\mathfrak{L}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{s}{a}\right), \quad \text{with ROC } R_1 = aR.} \tag{9.90}$$

a reversal about the $j\omega$-axis, together with a change in the size of the ROC by a factor of $|a|$. Thus, time reversal of $x(t)$ results in a reversal of the ROC. That is,

$$x(-t) \overset{\mathfrak{L}}{\longleftrightarrow} X(-s), \quad \text{with ROC} = -R. \tag{9.91}$$

Cont.

9.5.5 Conjugation

If

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s), \quad \text{with ROC} = R, \tag{9.92}$$

then

$$\boxed{x^*(t) \overset{\mathcal{L}}{\longleftrightarrow} X^*(s^*), \quad \text{with ROC} = R.} \tag{9.93}$$

Therefore,

$$X(s) = X^*(s^*) \quad \text{when } x(t) \text{ is real.} \tag{9.94}$$

Consequently, if $x(t)$ is real and if $X(s)$ has a pole or zero at $s = s_0$ (i.e., if $X(s)$ is unbounded or zero at $s = s_0$), then $X(s)$ also has a pole or zero at the complex conjugate point $s = s_0^*$.

Cont.

9.5.6 Convolution Property

If

$$x_1(t) \overset{\mathcal{L}}{\longleftrightarrow} X_1(s), \qquad \text{with } ROC = R_1,$$

and

$$x_2(t) \overset{\mathcal{L}}{\longleftrightarrow} X_2(s), \qquad \text{with } ROC = R_2,$$

then

$$\boxed{x_1(t) * x_2(t) \overset{\mathcal{L}}{\longleftrightarrow} X_1(s)X_2(s), \quad \text{with ROC containing } R_1 \cap R_2.} \tag{9.95}$$

Cont.

9.5.7 Differentiation in the Time Domain

If

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s), \qquad \text{with ROC} = R,$$

then

$$\boxed{\frac{dx(t)}{dt} \overset{\mathcal{L}}{\longleftrightarrow} sX(s), \qquad \text{with ROC containing } R.} \tag{9.98}$$

This property follows by differentiating both sides of the inverse Laplace transform as expressed in equation (9.56). Specifically, let

$$x(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} X(s)e^{st}ds.$$

Then

$$\frac{dx(t)}{dt} = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} sX(s)e^{st}ds. \tag{9.99}$$

Cont.

9.5.8 Differentiation in the s-Domain

Differentiating both sides of the Laplace transform equation (9.3), i.e.,

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt,$$

we obtain

$$\frac{dX(s)}{ds} = \int_{-\infty}^{+\infty} (-t)x(t)e^{-st}dt.$$

Consequently, if

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s), \qquad \text{with ROC} = R,$$

then

$$\boxed{-tx(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{dX(s)}{ds}, \qquad \text{with ROC} = R.} \tag{9.100}$$

Cont.

9.5.9 Integration in the Time Domain

If

$$x(t) \xleftrightarrow{\mathcal{L}} X(s), \qquad \text{with ROC} = R,$$

then

$$\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{\mathcal{L}} \frac{1}{s}X(s), \quad \text{with ROC containing } R \cap \{\mathcal{R}e\{s\} > 0\}. \tag{9.106}$$

This property is the inverse of the differentiation property set forth in Section 9.5.7. It can be derived using the convolution property presented in Section 9.5.6. Specifically,

$$\int_{-\infty}^{t} x(\tau)d\tau = u(t) * x(t). \tag{9.107}$$

From Example 9.1, with $a = 0$,

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}, \qquad \mathcal{R}e\{s\} > 0, \tag{9.108}$$

and thus, from the convolution property,

$$u(t) * x(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}X(s), \tag{9.109}$$

with an ROC that contains the intersection of the ROC of $X(s)$ and the ROC of the Laplace transform of $u(t)$ in eq. (9.108), which results in the ROC given in eq. (9.106).

TABLE 9.1 PROPERTIES OF THE LAPLACE TRANSFORM

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$ $x_1(t)$ $x_2(t)$	$X(s)$ $X_1(s)$ $X_2(s)$	R R_1 R_2
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re e\{s\} > 0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \to \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t \to \infty} x(t) = \lim_{s \to \infty} sX(s)$$

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\alpha$

Cont.

9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\alpha$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13	$[e^{-\alpha t}\cos \omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$

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