Cramer's Rule

Example

$$2x + 3y = 1$$
$$-2x + 4y = 2$$

Step-1

$$\begin{bmatrix} 2 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A \quad X = B$$

$$x = \frac{|A_x|}{|A|} \quad , \quad y = \frac{|A_y|}{|A|}$$

$$|A| = \begin{vmatrix} 2 & 3 \\ -2 & 4 \end{vmatrix} = 2(4) - (-2)(3)$$

$$= 8 + 6$$

$$\boxed{|A| = 14}$$

$$A = \begin{bmatrix} 2 & 3 \\ -2 & 4 \end{bmatrix}, \quad X \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Now,

$$A_x = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

$$= (1)(4) - (2)(3)$$

$$= 4 - 6$$

$$|A_x| = -2$$

$$A_y = \begin{bmatrix} 2 & 1 \\ -2 & 2 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 2 & 1 \\ -2 & 2 \end{vmatrix}$$

$$= (2)(2) - (-2)(1)$$

$$= 4 + 2$$

$$|A_y| = 6$$

Formula

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-2}{14}$$

$$= \frac{-2^{1}}{14_{7}} = \frac{-1}{7}$$

$$\boxed{x = \frac{-1}{7}}$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{6}{14} = \frac{6^{3}}{14_{7}}$$

$$\boxed{y = \frac{3}{7}}$$