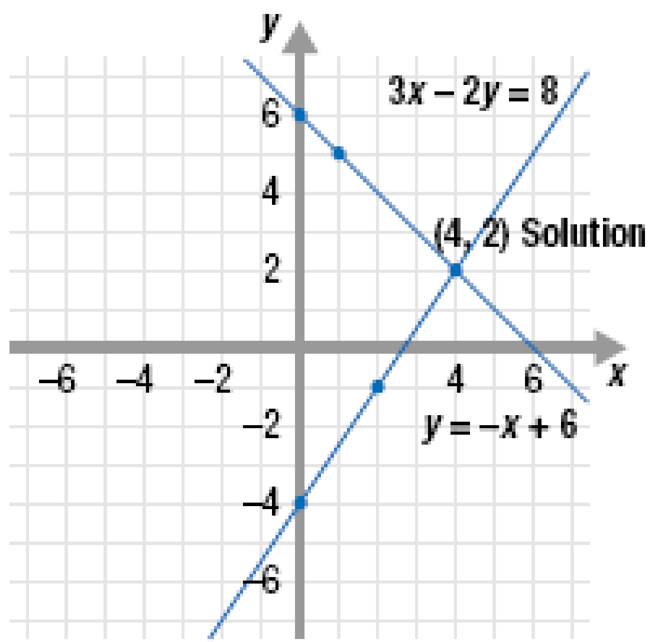
Solving Systems by Graphing and Substitution



The system has one solution, (4, 2).

$y = x + 1 \qquad 2y = 3x$

$2y = 3x$

$2(x + 1) = 3x$

$2x + 2 = 3x$

$-2x \qquad\qquad -2x$

$2 = x$

$y = x + 1$

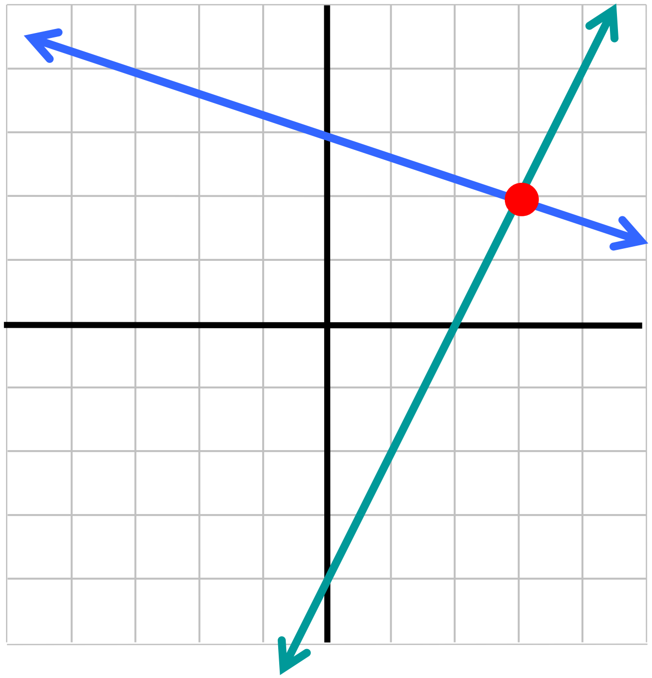
$y = 2 + 1 = 3$

Solution: (1, 3)

Solving Systems of Linear Equations by Graphing

$$\begin{cases} y = 2x - 4 \\ y = -\frac{1}{3}x + 3 \end{cases}$$

$$Solution: (3, 2)$$



$$2 = 2(3) - 4$$

$$2 = 2$$

$$2 = -\frac{1}{3}(3) + 3$$

$$2 = 2$$

Graphing to Solve a Linear System

Let's summarize! There are **4 steps** to solving a linear system using a graph.

Step 1: Put both equations in slope - intercept form.

Solve both equations for *y*, so that each equation looks like

$$y = mx + b.$$

Step 2: Graph both equations on the same coordinate plane.

Use the slope and *y* - intercept for each equation in step 1. Be sure to use a ruler and graph paper!

Step 3: Estimate where the graphs intersect.

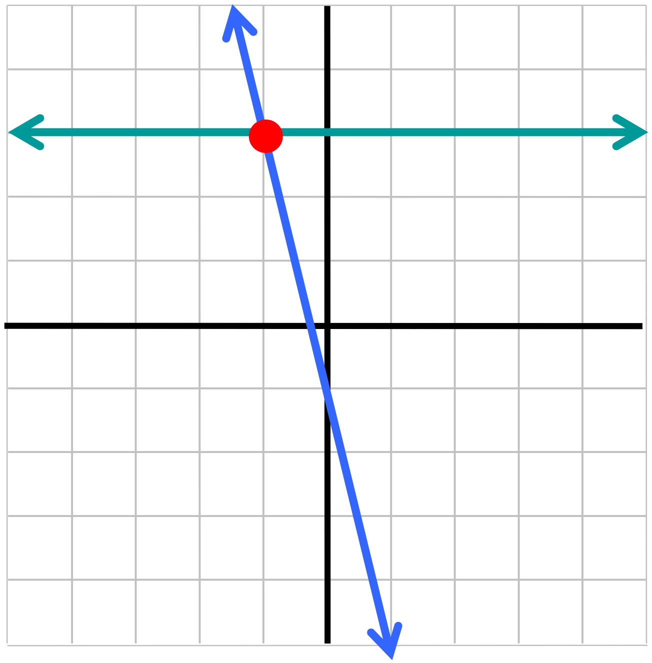
This is the solution! LABEL the solution!

Step 4: Check to make sure your solution makes both equations true.

Substitute the *x* and *y* values into both equations to verify the point is a solution to both equations.

Solving Systems of Linear Equations by Graphing

$$\begin{cases} y = 3 \\ y = -4x - 1 \end{cases}$$



$$Solution: (-1, 3)$$

$$3 = 3$$

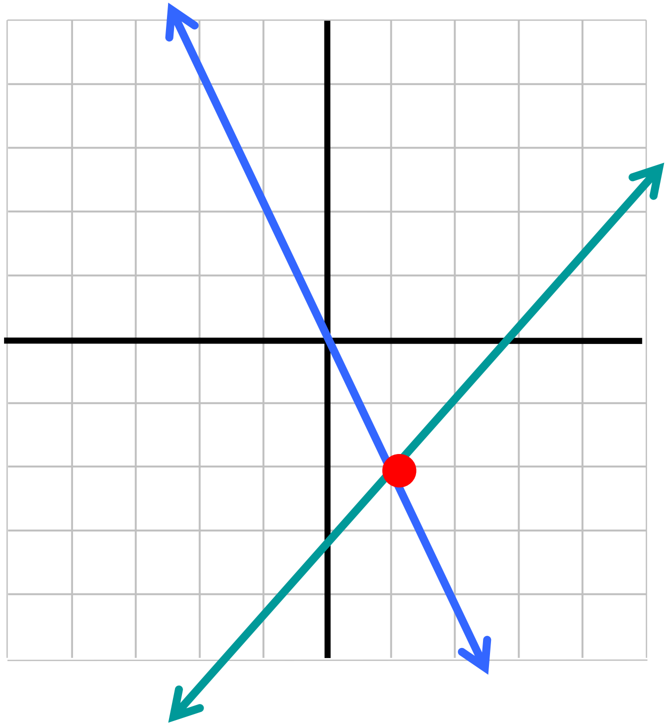
$$3 = -4(-1) - 1$$

$$3 = 3$$

Solving Systems of Linear Equations by Graphing

$$\begin{cases} x - y = 3 \\ 2x + y = 0 \end{cases}$$

$$\begin{cases} y = x - 3 \\ y = -2x \end{cases}$$



$$Solution: (1, -2)$$

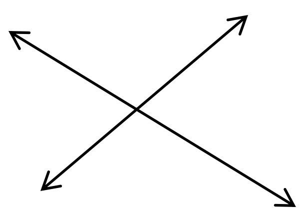
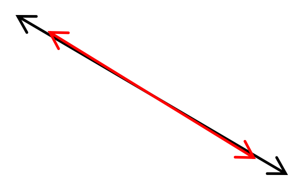
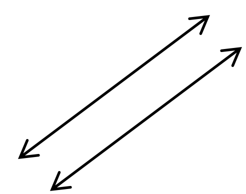
$$1 - (-2) = 3$$

$$2(1) + (-2) = 0$$

$$3 = 3$$

$$0 = 0$$

CLASSIFICATION OF LINEAR SYSTEMS (p.278)

Classification	Consistent and Independent	Consistent and Dependent	Inconsistent
Number of Solutions	Exactly One	Infinitely Many	None
Description	Different Slopes	Same Slope, Same y-intercept	Same Slope, Different y-intercept
Graph			

$Y=3x+4$ $Y=-3x+2$	Different slopes	**CONSISTENT and INDEPENDENT** so there is 1 solution to the system
$Y=1/2\ x + 10$ $Y = 1/3\ x + 10$	Different slopes	**CONSISTENT and INDEPENDENT** so there is 1 solution to the system
$Y = 4x + 5$ $Y = 4x + 5$	Same slope, Same y-intercept	**CONSISTENT and DEPENDENT** So there are infinite solutions
$Y = -3x + 1$ $Y = -3x - 1$	Same slope, Different y-intercepts	**INCONSISTENT** So there is no solution
$2y = 10x + 14$ $y = 5x + 7$	Not slope-intercept form Change the 1st equation to Y=5x+7, then Same Slope, Same y-intercept	**CONSISTENT and DEPENDENT** So there are infinite solutions

Example 1

x + 5y = 9 (1)

3x – 2y = 12 (2)

To solve, rewrite each equation in the form y = mx +b

<u>Isolating y in line (1)</u>

x + 5y = 9

5y = -x + 9

$$y = \frac{-x+9}{5}$$

$$\boxed{y = -\frac{1}{5}x + \frac{9}{5}}$$

<u>Isolating y in line (2)</u>

3x – 2y = 12

-2y = -3x + 12

$$y = \frac{-3x+12}{-2}$$

$$\boxed{y = \frac{3}{2}x - 6}$$

What type of system is it?

$$y = -\frac{1}{5}x + \frac{9}{5}$$

$$y = \frac{3}{2}x - 6$$

What is the slope and y-intercept for line (1)?

$$m = -\frac{1}{5}$$

$$b = \frac{9}{5}$$

What is the slope and y-intercept for line (2)?

$$m = \frac{3}{2}$$

$$b = -6$$

Since the lines have *different slopes* they will intersect. The system will have *one solution* and is classified as being consistent-independent.

Objective

The student will be able to:

solve systems of equations using substitution.

A-REI.3.6

Solving Systems of Equations

- You can solve a system of equations using different methods. The idea is to determine which method is easiest for that particular problem.
- These notes show how to solve the system algebraically using SUBSTITUTION.

Solving a system of equations by substitution

Step 1: Solve an equation for one variable.	Pick the easier equation. The goal is to get y= ; x= ; a= ; etc.
Step 2: Substitute	Put the equation solved in Step 1 into the other equation.
Step 3: Solve the equation.	Get the variable by itself.
Step 4: Plug back in to find the other variable.	Substitute the value of the variable into the equation.
Step 5: Check your solution.	Substitute your ordered pair into BOTH equations.

1) Solve the system using substitution

$$x + y = 5$$

$$y = 3 + x$$

Step 1: Solve an equation for one variable.

The second equation is already solved for y!

Step 2: Substitute

$$x + y = 5$$

$$x + (\mathbf{3 + x}) = 5$$

Step 3: Solve the equation.

$$2x + 3 = 5$$

$$2x = 2$$

$$x = 1$$

1) Solve the system using substitution

$$x + y = 5$$

$$y = 3 + x$$

Step 4: Plug back in to find the other variable.

$x + y = 5$

$(1) + y = 5$

$y = 4$

Step 5: Check your solution.

(1, 4)

$(1) + (4) = 5$ ✓

$(4) = 3 + (1)$ ✓

The solution is (1, 4). What do you think the answer would be if you graphed the two equations?

Which answer checks correctly?

$3x - y = 4$

$x = 4y - 17$

1. (2, 2)
2. (5, 3)
3. ✓ (3, 5)
4. (3, -5)

2) Solve the system using substitution

$$3y + x = 7$$

$$4x - 2y = 0$$

Step 1: Solve an equation for one variable.

It is easiest to solve the first equation for x.

$$3y + x = 7$$

$$-3y \qquad -3y$$

$$x = -3y + 7$$

Step 2: Substitute

$$4x - 2y = 0$$

$$4(-3y + 7) - 2y = 0$$

2) Solve the system using substitution

$$3y + x = 7$$
$$4x - 2y = 0$$

Step 3: Solve the equation.

$$-12y + 28 - 2y = 0$$
$$-14y + 28 = 0$$
$$-14y = -28$$
$$\mathbf{y = 2}$$

Step 4: Plug back in to find the other variable.

$$4x - 2y = 0$$
$$4x - 2(\mathbf{2}) = 0$$
$$4x - 4 = 0$$
$$4x = 4$$
$$\mathbf{x = 1}$$

2) Solve the system using substitution

$$3y + x = 7$$

$$4x - 2y = 0$$

Step 5: Check your solution.

(1, 2)

$3(2) + (1) = 7$ ✓

$4(1) - 2(2) = 0$ ✓

When is solving systems by substitution easier to do than graphing?

When only one of the equations has a variable already isolated (like in example #1).

If you solved the first equation for x, what would be substituted into the bottom equation.

$$2x + 4y = 4$$
$$3x + 2y = 22$$

1. $-4y + 4$
2. ✓ $-2y + 2$
3. $-2x + 4$
4. $-2y + 22$

3) Solve the system using substitution

$x = 3 - y$

$x + y = 7$

Step 1: Solve an equation for one variable.

The first equation is already solved for x!

Step 2: Substitute

$x + y = 7$

$(\mathbf{3 - y}) + y = 7$

Step 3: Solve the equation.

$3 = 7$

The variables were eliminated!!

This is a special case.

Does 3 = 7? FALSE!

When the result is FALSE, the answer is NO SOLUTIONS.

3) Solve the system using substitution

$2x + y = 4$

$4x + 2y = 8$

Step 1: **Solve an equation for one variable.**

The first equation is easiest to solved for y!

$y = -2x + 4$

Step 2: **Substitute**

$4x + 2y = 8$

$4x + 2(-2x + 4) = 8$

Step 3: **Solve the equation.**

$4x – 4x + 8 = 8$

$8 = 8$

This is also a special case.

Does 8 = 8? TRUE!

When the result is TRUE, the answer is **INFINITELY MANY SOLUTIONS**.