The line integral of B around a closed path is equal to $\mu_0$ times the total current through a closed path. The example of a long straight wire is frequently used. In which 'B' is at a distance 'r' from the conductor is

$$B = \frac{\mu_0 I}{2\pi r}$$

It is tangential to a circle of radius 'r' with center at the conductor. The current is directed upward and 'c' is described in counter clock wise.

$$B \cdot dl = |B||dl| \cos x = |B| r d\theta$$

putting the value of B in (b)

Eq.

$$\int_0^{2\pi} \frac{\mu_0 I}{2\pi \cancel{r}} \cancel{r} d\theta = \mu_0 \int_s \bar{J}(\vec{r_1}) \hat{n}\, da$$

$$\because l = r\theta$$

$$dl = r d\theta$$

$$\int_0^{2\pi} \frac{\mu_0 I}{2\pi} d\theta = \mu_0 \int_s \bar{J}(\vec{r_1}) \hat{n}\, da$$

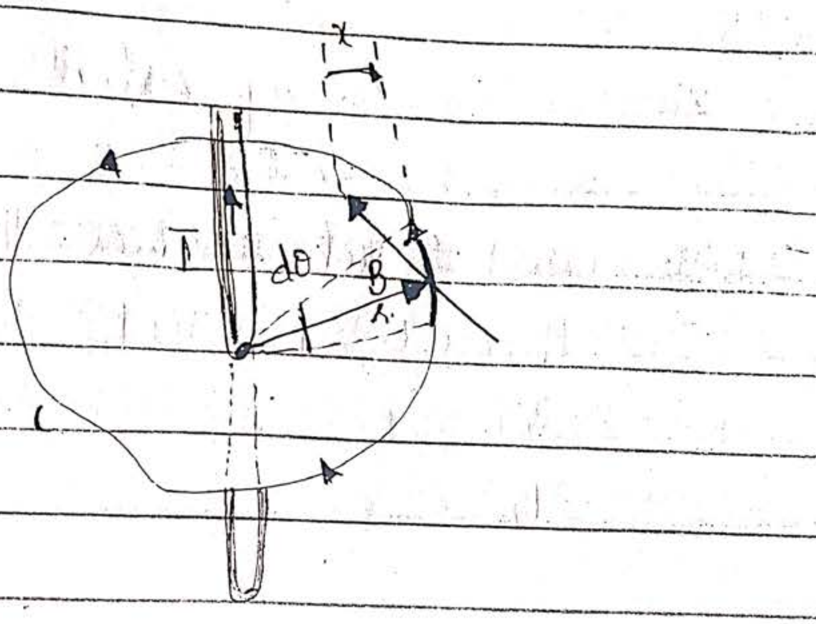
$$\frac{\mu_0 I}{2\pi} \int_0^{2\pi} d\theta = \mu_0 \int_s \bar{J}(\vec{r_1}) \hat{n}\, da$$

$$\frac{\mu_0 I}{2\pi}(2\pi - 0) = \mu_0 \int_S \bar{J}(\vec{r_1}) \cdot \hat{n}\, da.$$

$$\frac{\cancel{\mu_0} I (\cancel{2\pi})}{\cancel{2\pi}} = \mu_0 \int_S \bar{J}(\vec{r_1}) \cdot \hat{n}\, da$$

$$I = \int_S J(\vec{r_1}) \cdot \hat{n}\, da.$$

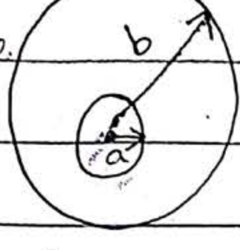
which is Integral form of Ampere's law.




Ampere's Circuital law for Long straight wire

Example # 2:-

consider a coaxial cable consisting of small center conductor of radius 'a' and 'a' coaxial cylindrical outer cable conductor of radius 'b'. Assume that these two conductors carry equal total current of Magnitude 'I', in opposite direction. The centers being directed out of the paper.




'B' Must be tangent to a circle centered on the center conductor. 'B' can't depend upon Azimuthal Angle.

For such a circle of radius 'r'

$$\oint B \cdot dl = 2\pi r B.$$

Which must equal $\mu_0$ times the total current through the loop. Thus

$$2\pi r B = \mu_0 I$$

If $a < r < b$

and

$$2\pi r B = 0$$

If $b < r$.

☆

## The Magnetic Vector Potential :-

The Magnetic Vector potential is represented by 'A'. The M-I curl doesnot vanishes, however, its divergence does. The divergence of Any curl is zero. i-e

$\nabla \cdot B = 0$

Because No monopoles exist.

Also curl $E = 0$

Because $E = -\text{grad}\, u$ which show Electric field changes.

Now we assume the M.I

$$\vec{B} = \text{curl}\, \vec{A}$$

or $$\vec{B} = \nabla \times A$$

By Taking curl of Above equation.

$$\text{curl}\, \vec{B} = \text{curl}\, \text{curl}\, \vec{A}$$

$$\text{curl}\, \vec{B} = \nabla \times (\nabla \times \vec{A})$$

By using vector Identity

$$\nabla \times (\nabla \times A)\ \text{curl}\, \vec{B} = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{A}) \mp \nabla^2 \vec{A}$$

$$\text{curl}\, \vec{B} = + \nabla^2 \vec{A}$$

Also

$$-\mu_0 \vec{J} = + \nabla^2 \vec{A}$$

$\because \nabla \cdot \vec{A} = 0$ Divergence of 'A' is zero

Now By using Poisson's Equation.

(chang in potential) $\nabla^2 U = -\rho / \varepsilon_0$

The solution of P.E is

$$U = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r_1)}{r_2 - r_1} \, dv$$

Similarly we can write as

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r_1})}{(\vec{r_2}-\vec{r_1})} dv_1$$

$$\vec{A}(r) = \frac{\mu_0}{4\pi}\int_v \frac{\vec{J}(r_1)}{(\vec{r_2}-\vec{r_1})} dv_1$$

This Method is Easier to find than Biot Savart's law.

The alternative Method to find is the direct transformation of

$$B(r_2) = \frac{\mu_0}{4\pi}\int_v \vec{J}(\vec{r_1}) \times \frac{(r_2 - r_1)}{|\vec{r_2}-\vec{r_1}|^3} dv_1$$

we can write As.

$$\frac{\vec{r_2}-\vec{r_1}}{|\vec{r_2}-\vec{r_1}|^3} = -(\text{grad})_2 \frac{1}{|\vec{r_2}-\vec{r_1}|} = -\nabla_2 \frac{1}{|\vec{r_2}|}$$

'$\nabla_2$' Indicates differentiation w.r.t '$r_2$'

Taking curl of $\frac{J(\vec{r_1})}{(\vec{r_2}-\vec{r_1})}$

$$\text{curl}_2 \frac{\vec{J}(\vec{r_1})}{(\vec{r_2}-\vec{r_1})} = -\vec{J}(\vec{r_1}) \times \text{grad}_2 \frac{1}{\vec{r_2}-\vec{r_1}}$$

$$+ \frac{1}{\vec{r_2}-\vec{r_1}} \text{curl}_2 \vec{J}(\vec{r_1})$$

The 2nd term vanishes so Eq becomes.

B1

$$\text{curl}_2 \frac{\overline{J}(\vec{r}_1)}{\vec{r}_2 - \vec{r}_1} = -\overline{J}(r_1) \, \text{grad}_2 \frac{1}{\vec{r}_2 - \vec{r}_1}$$

put this in Integral

$$B(\vec{r}_2) = \frac{\mu_0}{4\pi} \int_V J(r_1) \times \nabla_2 \frac{1}{|\vec{r}_2 - \vec{r}_1|} \, dv_1$$

$$B(\vec{r}_2) = \frac{\mu_0}{4\pi} \int_V \nabla_2 \times \frac{J(r_1)}{|\vec{r}_2 - \vec{r}_1|} \, dv_1$$

curl $\text{curl} \left[ \frac{\mu_0}{4\pi} \int \frac{J(r_1)}{r_2 - r_1} dv_1 \right]$

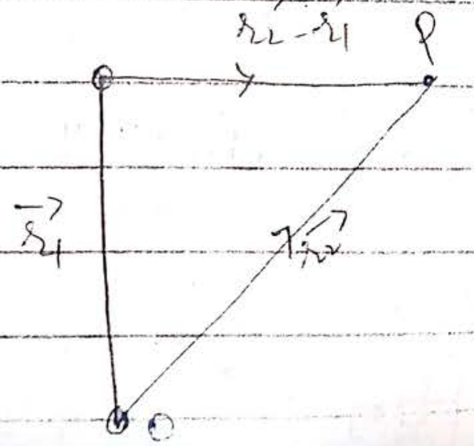
As $B = \text{curl} \, \vec{A}$

$$\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{J(r_1) \, dv_1}{(r_2 - r_1)}$$

$B = \text{curl}[A]$

which shows the Magnetic vector potential. It should be noted that evaluating the vector potential at a single point is not usefull, becau The M.I is obtained by differentiation.

# The Magnetic Field Of A Distant Circuit:-



The magnetic vector potential due to small circuit at a large distances be evaluated easily. we can Apply Magnetic vector potential to find of a distant circuit.

$$\vec{A}(r_2) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(r_1)}{|r_2 - r_1|}\, dv_1$$

As By substitution

$$J(r_1)\,dv_1 = I\,dr_1$$

$$\vec{A}(\vec{r_2}) = \frac{\mu_0}{4\pi}\int \frac{I\,dr_1}{|r_2 - r_1|}$$

$$A(\vec{r_2}) = \frac{\mu_0 I}{4\pi}\int \frac{dr_1}{|r_2 - r_1|} \longrightarrow (1)$$

For circuits whose dims are small compared $r_2$ the denominator can be approximated

Now. By using Approximation to solve the denominator.

$$[(\vec{r_2} - \vec{r_1})^2]^{-1/2} = [r_2^2 + r_1^2 - 2r_1 \cdot r_2]^{-1/2}$$

Expand in powers of $r_1/r_2$ to get

$$|r_2 - r_1|^{-1/2} = \frac{1}{r_2^2}\left[1 + \frac{r_1 \cdot r_2}{r_2^2}\right]$$

Now By Neglecting the higher power.

Now $A(\vec{r_2})$

$$A(\vec{r_2}) = \frac{\mu_0 I}{4\pi}\left[\frac{1}{r_2}\oint dr_1 + \frac{1}{r_2^3}\oint dr_1\,(\vec{r_1}\cdot\vec{r_2}) + \cdots\right] \rightarrow$$

we need only one Integral so we apply Identity $\cdot - r_1(r_2 \cdot dr_1) + dr_1(r_1 \cdot r_2)$

$$(r_1 \times dr_1) \times r_2 = \cancel{dr_1(r_1 \cdot r_2)} - \cancel{r_2(r_1 \cdot dr_1)} \rightarrow 3a$$

$$\text{or } (A \times B) \times C = B(A \cdot C) - C(A \cdot B) \;\therefore -C(A \cdot B) + B(A \cdot C)$$

Now this is also the vector Identity

$$d\left[\vec{r_1} \cdot (\vec{r_2} \cdot \vec{r_1})\right] = r_1(r_2 \cdot dr_1) + dr_1(r_2 \cdot r_1) \rightarrow 4$$

$$\therefore r_2(r_1 \cdot dr_1) = r_1(r_2 \cdot dr)$$

By Adding equation $(3^a)$ and $(4)$.

$$d[\vec{r_1} \cdot (\vec{r_2} \cdot \vec{r_1})] + (r_1 \times dr_1) \times r_2 = dr_1(r_1 \cdot r_2) - r_2(\cancel{r_1} \cdot dr_1) + r_2(\cancel{r_1} \cdot dr_1) + dr_1 (r_2 \cdot \cancel{r_1})$$

$$d[\vec{r_1} \cdot (\vec{r_2} \cdot \vec{r_1})] + (\vec{r_1} \times dr_1) \times r_2 = 2 dr_1 (r_2 \cdot r_1)$$

we can also write as

$$dr_1(\vec{r_2} \cdot \vec{r_1}) = \frac{1}{2}\left[d(\vec{r_1} \cdot (\vec{r_2} \cdot \vec{r_1})) + (\vec{r_1} \times d\vec{r_1})\right.$$

diff + integral = 0.

The first term on the R.H.S is exact differential so It does not contribute.

$$dr_1(\vec{r_2} \cdot \vec{r_1}) = \frac{1}{2}\left[(\vec{r_1} \times dr_1) \times \vec{r_2}\right]$$

$$A(\vec{r_2}) = \frac{\mu_0 I}{4\pi} \frac{1}{r_2^3}\left[\frac{1}{2}\oint (\vec{r_1} \times dr_1) \times r_2\right]$$

$$A(\vec{r_2}) = \frac{\mu_0 I}{4\pi} \frac{\vec{r_2}}{r_2^3}\left[\frac{1}{2}\oint (\vec{r_1} \times d\vec{r_1})\right] \times \frac{\vec{r_2}}{|r_2|^3}$$

Property of magnet that interacts with an applied field to give a mechanical moment

$$A(\vec{r_2}) = \frac{\mu_0}{4\pi}\left[\frac{I}{2}\oint \vec{r_1} \times d\vec{r_1}\right] \times \frac{\vec{r_2}}{r_2^3} \rightarrow (5)$$

In Equation (5) the term in the brackets shows the "Magnetic Moment represented by 'm' of the circuit.

$$A(\vec{r_2}) = \frac{\mu_0}{4\pi}\,\frac{\vec{m} \times r_2}{r_2^3}$$

In this derivation It has been Assumed that all $r_1 << r_2$ hence The above Equation is not valid for an arbitrary region, but only for the origin close to the circuit.

The M.I can be determined by taking curl of Equation

$$B(\vec{r_2}) = \text{curl}\, A(\vec{r_2}) = \text{curl}\left\{\frac{\mu_0}{4\pi}\,\frac{\vec{m} \times \vec{r_2}}{\vec{r_2}^3}\right\}$$

$$B(\vec{r_2}) = \frac{\mu_0}{4\pi}\,\text{curl}\left(\overset{F}{\vec{m}} \times \overset{G}{\frac{\vec{r_2}}{r_2^3}}\right)$$

Now by Applying the vector Identity.

$$\text{curl}\,(\vec{F} \times \vec{G}) = \vec{F}\,\text{div}\,G - G\,\text{div}\,F + (\vec{G} \cdot \nabla)\vec{F} - (\vec{F} \cdot \nabla)\vec{G}$$

gradient is rate at which something changes over a distance or time;

35

So eq becomes.

$$B(\vec{r_2}) = \frac{\mu_0}{4\pi}\left[ m \, div \frac{\vec{r_2}}{r_2^3} - \frac{\vec{r_2}}{r_2^3} div \vec{m} + \left(\frac{\vec{r_2}}{r_2^3} \cdot grad\right) \vec{m} - (\vec{m} \, grad) \frac{\vec{r_2}}{r_3^2} \right]$$

as uniform magnetic field

Here $grad \; \vec{m} = 0$

Also $div \; \vec{m} = 0$ — monopole doesn't exist

$\nabla . B = 0$
$\Rightarrow \nabla . m = 0$ ] as field can't be diverged

so

$$B(\vec{r_2}) = \frac{\mu_0}{4\pi}\left[ m \, div \frac{\vec{r_2}}{r_2^3} - (\vec{m} \, grad) \frac{\vec{r_2}}{r_2^3} \right]$$

As we know that

$$div(\vec{U} \cdot \vec{V}) = U \, div \, \vec{V} - \vec{V} \, grad \, U$$

So

$$div \frac{\vec{r_2}}{r_2^3} = \frac{1}{r_2^3} div \, \vec{r_2} - \vec{r_2} \, grad \frac{1}{r_2^3}$$

$\because div(r_2 - r_1)$, $|r_2 - r_1| = grad\left(\frac{1}{r_2 - r_1}\right)$

* $\frac{div(\vec{r_2} - \vec{r_1})}{|r_2 - r_1|} \quad grad\left(\frac{1}{r_2 - r_1}\right)$

$$\vec{B}(\vec{r_2}) = curl \, A(\vec{r_2}) = -\frac{\mu_0}{4\pi}\left[ \vec{m} \, grad \frac{\vec{r_2}}{r_2^3} \right]$$

Now we take

$$\vec{m} = m_x \hat{i} + m_y \hat{j} + m_z \hat{k}$$

$$\vec{m} \, grad = \left[ m_x \frac{\partial}{\partial x} + m_y \frac{\partial}{\partial y} + m_z \frac{\partial}{\partial z} \right]$$

Put this value in above Integral

$$B(\vec{r_2}) = \text{curl } A(\vec{r_2}) = -\frac{\mu_0}{4\pi}\left[m_x \frac{\partial}{\partial x_2} + m_y \frac{\partial}{\partial y_2} + m_z \frac{\partial}{\partial z_2}\right]\frac{\vec{r_2}}{r_2^3}$$

$$\left(\vec{r}^2 = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}\right)$$

$$m_x \frac{\partial}{\partial x_2}\left(\frac{\vec{r_2}}{r_2^3}\right) = m_x \frac{\partial}{\partial x_2}(x_2^2 + y_2^2 + z_2^2)^{-3/2} \cdot (x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$$

$$m_x \frac{\partial}{\partial x_2}\left(\frac{\vec{r_2}}{r_2^3}\right) = m_x\left[-\frac{3}{\cancel{2}} \cdot \cancel{2}x(x_2^2 + y_2^2 + z_2^2)^{-5/2} \cdot \vec{r_2} + (1) \cdot \frac{1}{r_2^3}\right]$$

$$\frac{\partial y}{\partial x} = \frac{\partial z}{\partial x} = 0$$

$$m_x \frac{\partial}{\partial x_2}\left(\frac{\vec{r_2}}{r_2^3}\right) = m_x\left[-3x(x_2^2 + y_2^2 + z_2^2)^{-5/2} \cdot \vec{r_2} + 1/r_2^3\right]$$

$$m_x \frac{\partial}{\partial x_2}\left(\frac{\vec{r_2}}{r_2^3}\right) = -3m_x x (r_2^2)^{-5/2} \cdot \vec{r_2} + \frac{m_x}{r_2^3}$$

$$m_x \frac{\partial}{\partial x_2}\left(\frac{\vec{r_2}}{r_2^3}\right) = -3x m_x r_2^{-5} \cdot \vec{r_2} + \frac{m_x}{r_2^3}$$

By Generallizing 'x' replace by vector '$r_2$'

$$m_x \frac{\partial}{\partial x_2}\left(\frac{\vec{r_2}}{r_2^3}\right) = -\frac{m_x}{r_2^3} + \frac{3(m_x \cdot r_2) \cdot r_2}{r_2^5}$$

(Magnetic Dipole)

$$B(r_1) = \frac{\mu_0}{4\pi}\left[-\frac{m_x}{r_2^{3}} + \frac{3 m_x r_2 \, r_2}{r_1^{-5}}\right] \quad 19 \qquad 37$$

This Equation shows the M.F of a distant circuit does not depend on its detailed geometry but only its Magnetic Moment $\vec{m}$. This Eq of the same form as the E.F due to an Electric dipole which Explain the same Magnetic dipole field, $\vec{m}$ is usually called the "Magnetic dipole Moment of the circuit".

## The Magnetic Scalar Potential :-

$\nabla \times B = \mu_0 J$ (.44)

The div $B = 0$ but curl $\vec{B} = \mu_0 \vec{J} = 0$ when current density is zero. Scalar is always included with gradiant. If we have a scalar $U$ then the gradiant of a scalar potential is

change in pot = current

$$\vec{B} = -\mu_0 \text{ grad } U \quad \longrightarrow (I) \text{ Produced.}$$

In Actual $U$ is the P.E Which is changing and is decreasing so '−' sign is included.

As the divergence of $\vec{B}$ is also zero so,

$$\text{div } \vec{B} = -\mu_0 (\text{div grad } U) = 0$$