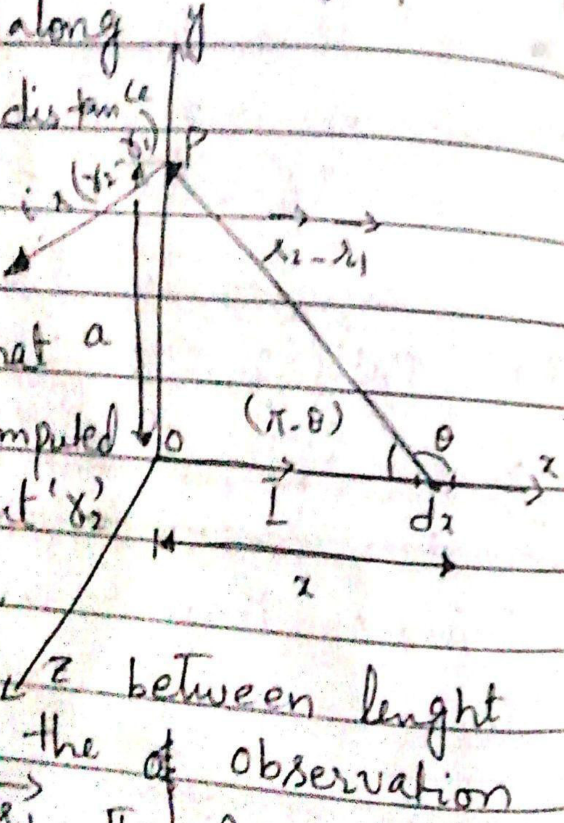
Applications On Biot-Savart Law.

1)

ON A Long Straight wire:-

In this case we considered the magnetic field due to a long straight wire. The long straight wire is taken along x-axis from minus infinity to plus infinity and carry a current 'I' which is flowing from origin.

A point is taken along y-axis is 'P'. The distance from origin to 'P' is 'a'. In short we can say that a field will be computed at a typical point 'r_2' on the y-axis.



The distance between lenght element and the of observation point is $\vec{r_2}-\vec{r_1}$. The Angle is θ.

Now the magnetic Induction is

$$B(r_2) = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{+\infty}\frac{dx\,\hat{i}\times(r_2-r_1)}{|r_2-r_1|^3}$$

Since $r_2 - r_1$ lies in xy-plane. So,

$$\hat{i} \times (\vec{r}_2 - \vec{r}_1) = |\vec{r}_2 - \vec{r}_1| \sin\theta \hat{k}$$

The direction is along x-axis So $i = 1$

Now from figure.

$$\frac{Per}{Base} = \tan(\pi - \theta)$$

Per = a ; Base = x ; Hyp = $\vec{r}_2 - \vec{r}_1$

$$\frac{a}{x} = -\tan\theta$$

$$a = -x\tan\theta$$

or $$x = \frac{-a}{\tan\theta}$$

So $\because \cot\theta = 1/\tan\theta$

$$x = -a\cot\theta$$

$\because \cot\theta = \frac{\cos\theta}{\sin\theta}$

$$x = \frac{-a\cos\theta}{\sin\theta}$$

$$\boxed{dx = -a\,\text{cosec}^2\theta}$$ $\because$

Also

$$\text{Cosec}(\pi-\theta) = \frac{Hyp}{Per} = \frac{\vec{r}_2 - \vec{r}_1}{a}$$

$$a\,\text{cosec}\,\theta = r_2 - r_1$$