Ampere's Circuital Law:-

If the steady current is flowing then current density is

$$\bar{J} = \frac{I}{A}$$

$\nabla \times \bar{J}$ = curl
$\nabla \cdot \bar{J}$ = Divergence
$\nabla \bar{J}$ = Gradient

And Magnetic Induction is to find.

$$\nabla \cdot \bar{J} = 0$$

As we know the value of M-I is

$$B(r_2) = \frac{\mu_0}{4\pi} \int_V J(r_1) \times \frac{(r_2 - r_1)}{|r_2 - r_1|^3} dv_1$$

Taking curl w.r.t r_2 of above Equation.

$$\nabla_2 \times (B r_2) = \frac{\mu_0}{4\pi} \vec{\nabla_2} \times \int_V J(r_1) \times \frac{(r_2 - r_1)}{|r_2 - r_1|^3} dv_1$$

By Applying the vector Identity.

$$\nabla_2 \times B(r_2) = \frac{\mu_0}{4\pi} \int_V \left[J(\vec{r_1}) \left(\nabla_2 \cdot \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|^3} \right) - \nabla_2 \vec{J}(\vec{r_1}) \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|^3} \right] dv_1$$

3 dimensional dirac delta fn

Now By Changing in solid Angle in first term.

$$\nabla_2 \times B(r_2) = \frac{\mu_0}{4\pi} \int_V \left[J(\vec{r_1}) 4\pi \delta (\vec{r_2} - r_1) - \nabla_2 \cdot \vec{J}(\vec{r_1}) \cdot \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|^3} \right] dv_1$$

$* \cdot \frac{1}{4\pi} \nabla \frac{1}{|r-r'|}$, delta fn in 3D: $\delta(r-r') = \frac{1}{4\pi} \nabla^2 \frac{1}{(r-r')}$

4π comes about from wie the divergence theorem to conver

The 2nd term vanishes because of steady current $\nabla_2 \cdot \vec{J}(\vec{r_1}) = 0$

$$\frac{\vec{\nabla_2} \cdot \vec{J}(\vec{r_1})\,(\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^3} = \frac{(\vec{r_2} - \vec{r_1})\,\nabla_2 \cdot \vec{J}(r_1)}{|r_2 - r_1|^3} + \vec{J}(\vec{r_1}) \cdot \nabla_2 \frac{r_2 - r_1}{|r_2 - r_1|}$$

Remaning term is

$$\nabla_2 \times B(\vec{r_2}) = \frac{\mu_0}{4\pi} \int_V \vec{J}(\vec{r_1})\, 4\pi \delta(r_2 - r_1)\, dv_1$$

$$\nabla_2 \times B(\vec{r_2}) = \frac{\mu_0}{\cancel{4\pi}} \times \cancel{4\pi} \int_V \vec{J}(\vec{r_1})\, \delta(r_2 - r_1)\, dv_1$$

Volume integral over all space in surface integral with 4π would be total solid angle.

$$\nabla_2 \times B(\vec{r_2}) = \mu_0 \int_V \vec{J}(\vec{r_1})\, \delta(r_2 - r_1)\, dv_1$$

$$\nabla_2 \times B(\vec{r_2}) = \mu_0 \vec{J}(r_1) \quad \rightarrow (a)$$

Which is the differential form of Ampere's law.

We can use Stoke's Theorem to convert this differential into an integral form

As Stoke's Theorem is

$$\int \nabla \times B \cdot n\, da = \oint \vec{B} \cdot dl$$

Now take the surface Integral of Eq (a)

$$\int_S \vec{\nabla_2} \times \vec{B} \cdot \hat{n}\, da = \mu_0 \int_S \vec{J}(r_1)\, \hat{n}\, da$$

Now by Stoke's Theorem

$$\oint B \cdot dl = \mu_0 \int \vec{J}(\vec{r_1})\, \hat{n}\, dl \rightarrow (b)$$