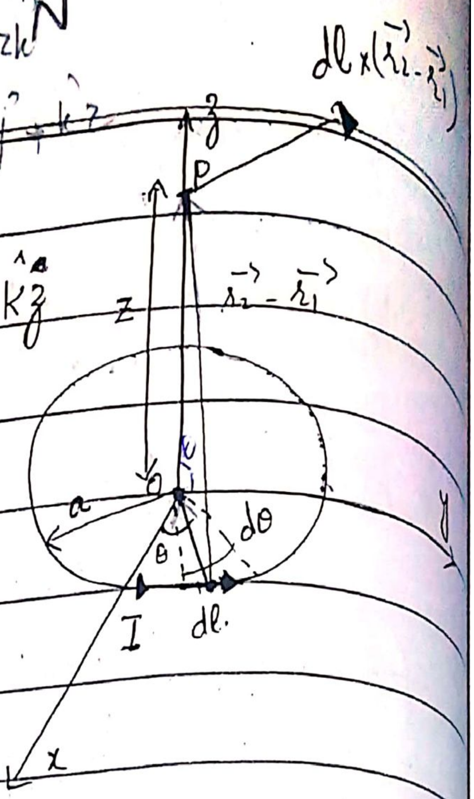
$r_1 = a\cos\theta\,\hat{i} + a\hat{j}\sin\theta$

$r_2 = \cancel{a\cos\theta\,\hat{i}}\ \cancel{a\sin\theta\,\hat{j}} + z\hat{k}$ N

$r_2 - r_1 = -a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j} + \hat{k}z$



$\vec{dl} = a\,d\theta\,(-\hat{i}\sin\theta + \hat{j}\cos\theta)$

$\vec{r_2} - \vec{r_1} = -\hat{i}a\cos\theta - \hat{j}a\sin\theta + \hat{k}z$

By Taking Magnitude

$|\vec{r_2} - \vec{r_1}| = (a^2 + z^2)^{1/2}$

Now The Magnetic Induction is

$$\vec{B}(\cancel{z_2}) = \frac{\mu_0 I_1}{4\pi} \oint \frac{\vec{dl_1} \times (\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^3}$$

$$\vec{B}(\cancel{z_2}) = \frac{\mu_0}{4\pi} I_1 \oint_0^{2\pi} \frac{\vec{dl_1} \times (\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^3}$$

For $\vec{dl_1} \times (\vec{r_2} - r_1)$ Taking determinant.

$$\vec{dl_1} \times (\vec{r_2} - \vec{r_1}) = \begin{vmatrix} i & j & K \\ -a\sin\theta d\theta & a\cos\theta d\theta & 0 \\ -a\cos\theta & a\sin\theta & z \end{vmatrix}$$

$$\vec{dl_1} \times (\vec{r_2} - \vec{r_1}) = \hat{i}(a\cos\theta\, z) + (a\sin\theta\, z)\hat{j} + K(a^2)(\sin^2\theta + \cos^2\theta)$$

$$\vec{dl_1} \times (\vec{r_2} - \vec{r_1}) = a\cos\theta z\hat{i} + a\sin\theta z\hat{j} + a^2 K$$

put this value in integral

$$\vec{B}(r_2) = \frac{\mu_0}{4\pi} I \oint_0^{2\pi} \frac{\hat{i}(a\cos\theta z) + (a\sin\theta z)\hat{j} + \hat{k}a^2}{(z^2 + a^2)^{3/2}}$$

By Integrating first two Terms

Equal to zero.

$$\vec{B}(z) = \frac{\mu_0 I}{4\pi} \oint_0^{2\pi} \frac{\hat{K} a^2}{(z^2+a^2)^{3/2}} \, d\theta$$

By Taking constant outside the Integral.

$$\vec{B}(z) = \frac{\mu_0 I}{4\pi} \frac{\hat{K} a^2}{(z^2+a^2)^{3/2}} \oint_0^{2\pi} d\theta$$

$$\vec{B}(z) = \frac{\mu_0 I}{4\pi} \cdot \frac{a^2}{(z^2+a^2)^{3/2}} \hat{K} \, (2\pi - 0)$$

$$\vec{B}(z) = \frac{\mu_0 I \cdot a^2}{\cancel{4\pi}_{2} (z^2+a^2)^{3/2}} \hat{K} \, (\cancel{2\pi})$$

$$\vec{B}(z) = \frac{\mu_0 I}{2} \cdot \frac{a^2}{(z^2+a^2)^{3/2}} \hat{K},$$

which is of course, entirely along z-axis

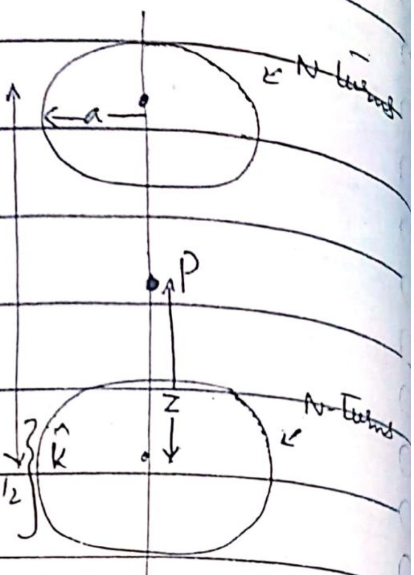
ON Helmholtz Coil:-

"Two coils which are at certain distance is known as Helmholtz coil".

A frequently used current configuration is the helmholtz coil which consists of two circular ~~coils~~ coils of the same radius, with a common axis, separated by a distance to make second derivative of 'B' vanish at point on the axis half way b/w the coils. As shown in figure.

As know the M.I of a circular wire

$$B(z) = \frac{\mu_0 I a^2}{2(z^2+a^2)^{3/2}} \hat{k}$$



Now M.I at Point 'p' is

$$B_z(z) = \frac{N\mu_0 I a^2}{2} \left\{ \frac{1}{(z^2+a^2)^{3/2}} + \frac{1}{[(2b-z)^2+a^2]^{3/2}} \right\} \hat{k}$$

Now Take differential w.r.t 'z'.

$$\frac{dB_z(z)}{dz} = \frac{N\mu_0 I a^2}{2} \left\{ \frac{-3}{\cancel{2}} \cdot \frac{\cancel{2}z}{(z^2+a^2)^{5/2}} + \frac{(-)3}{\cancel{2}} \cdot \frac{\cancel{2}(-(2b-z))}{[(2b-z)^2+a^2]^{5/2}} \right\} \hat{k}$$

$$\frac{dB_z(z)}{dz} = \frac{\mu_0 N I a^2}{2} \left\{ \frac{-3z}{(z^2+a^2)^{5/2}} \mp \frac{3(z-2b)}{[(2b-z)^2+a^2]^{5/2}} \right\} \hat{k}$$

$$\frac{dB_z(z)}{dz} = \frac{-3\mu_0 N I a^2}{2} \left\{ \frac{z}{(z^2+a^2)^{5/2}} \pm \frac{(z-2b)}{[(2b-z)^2+a^2]^{5/2}} \right\} \hat{k}$$

Now Taking 2nd Derivative of above Integral.

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 N I a^2}{2} \left\{ \frac{1}{(z^2+a^2)^{5/2}} + \left(\frac{-5}{\cancel{2}}\right) \cdot \frac{\cancel{2}z^2}{(z^2+a^2)^{7/2}} \right.$$

$$+ \frac{1}{\quad\quad\quad^{2}\ ^{2})^{5/2}} + (-)\left(\frac{5}{\cancel{2}}\right) \cdot \frac{\cancel{2}(z-2b)^2}{\quad\quad\quad)^{7/2}}$$

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$$\frac{d^2B(z)}{dz^2} = \frac{3\mu_0 N I a^2}{2}\left\{\frac{1}{(z^2+a^2)^{5/2}} - \frac{5z^2}{(z^2+a^2)^{7/2}} + \frac{1}{[(2b-z)^2+a^2]^{5/2}}\right.$$

$$\left. - \frac{5(z-2b)^2}{[(2b-z)^2+a^2]^{7/2}}\right\}$$

Put $z = b$.

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 N I a^2}{2}\left\{\frac{1}{(b^2+a^2)^{5/2}} - \frac{5b^2}{(b^2+a^2)^{7/2}} + \frac{1}{[(2b-b$$

$$\left. - \frac{5(b-2b)^2}{[(2b-b)^2+a^2]^{7/2}}\right\}$$

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 N I a^2}{2}\left\{\frac{1}{(b^2+a^2)^{5/2}} - \frac{5b^2}{(b^2+a^2)^{7/2}} + \frac{1}{(b^2+a^2)^{5/2}}\right.$$

$$\left. - \frac{5(-b)^2}{[b^2+a^2]^{7/2}}\right\}$$

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 N I a^2}{2}\left\{\frac{1}{(b^2+a^2)^{5/2}} - \frac{5b^2}{(b^2+a^2)^{7/2}} + \frac{1}{(b^2+a^2)^{5/2}}\right.$$

$$\left. - \frac{5b^2}{(b^2+a^2)^{7/2}}\right\}$$

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 N I a^2}{2}\left\{\frac{b^2+a^2-5b^2+b^2+a^2-5b^2}{(b^2+a^2)^{7/2}}\right\}$$

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 N I a^2}{2}\left\{\frac{2a^2-8b^2}{(b^2+a^2)^{7/2}}\right\}$$

$$\frac{d^2B(z)}{dz^2} = -\frac{3\mu_0 N I a^2}{\cancel{2}}\,\cancel{2}\left[\frac{(a^2-4b^2)}{(b^2+a^2)^{7/2}}\right]$$