Chapter One

INTRODUCTION

OCCURRENCE OF PLASMAS IN NATURE 1.1

It has often been said that 99% of the matter in the universe is in the plasma state: that is, in the form of an electrified gas with the atoms dissociated into positive ions and negative electrons. This estimate may not be very accurate, but it is certainly a reasonable one in view of the fact that stellar interiors and atmospheres, gaseous nebulae, and much of the interstellar hydrogen are plasmas. In our own neighborhood, as soon as one leaves the earth's atmosphere, one encounters the plasma comprising the Van Allen radiation belts and the solar wind. On the other hand, in our everyday lives encounters with plasmas are limited to a few examples: the flash of a lightning bolt, the soft glow of the Aurora Borealis, the conducting gas inside a fluorescent tube or neon sign, and the slight amount of ionization in a rocket exhaust. It would seem that we live in the 1% of the universe in which plasmas do not occur naturally.

The reason for this can be seen from the Saha equation, which tells us the amount of ionization to be expected in a gas in thermal equilibrium:

$$\frac{n_i}{n_n} \approx 2.4 \times 10^{21} \frac{T^{3/2}}{n_i} e^{-U_i/KT} \qquad [1\text{-}1]$$

Here n_i and n_n are, respectively, the density (number per m^3) of ionized atoms and of neutral atoms, T is the gas temperature in °K, K is Boltzmann's constant, and U_i is the ionization energy of the gas—that collision of high enough energy to knock out an electron. In a cold gas, such energetic collisions occur infrequently, since an atom must be accelerated to much higher than the average energy by a series of "favorable" collisions. The exponential factor in Eq. [1-1] expresses the fact that the number of fast atoms falls exponentially with U_i/KT. Once an atom is ionized, it remains charged until it meets an electron; it then very likely recombines with the electron to become neutral again. The recombination rate clearly depends on the density of electrons, which we can take as equal to n_i. The equilibrium ion density, therefore, should decrease with n_i; and this is the reason for the factor n_i^{-1} on the right-hand side of Eq. [1-1]. The plasma in the interstellar medium owes its existence to the low value of n_n (about 1 per cm^3), and hence the low recombination rate.

DEFINITION OF PLASMA 1.2

Any ionized gas cannot be called a plasma, of course; there is always some small degree of ionization in any gas. A useful definition is as follows:

A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior. due to the long range coulomb force

We must now define "quasineutral" and "collective behavior." The meaning of quasineutrality will be made clear in Section 1.4. What is meant by "collective behavior" is as follows.

Consider the forces acting on a molecule of, say, ordinary air. Since the molecule is neutral, there is no net electromagnetic force on it, and the force of gravity is negligible. The molecule moves undisturbed until it makes a collision with another molecule, and these collisions control the particle's motion. A macroscopic force applied to a neutral gas, such as from a loudspeaker generating sound waves, is transmitted to the individual atoms by collisions. The situation is totally different in a plasma, which has *charged* particles. As these charges move around, they can generate local concentrations of positive or negative charge, which give rise to electric fields. Motion of charges also generates currents, and hence magnetic fields. These fields affect the motion of other charged particles far away.

Let us consider the effect on each other of two slightly charged regions of plasma separated by a distance r (Fig. 1-1). The Coulomb force between A and B diminishes as $1/r^2$. However, for a given solid angle (that is, $\Delta r/r$ = constant), the volume of plasma in B that can affect

A increases as r^3. Therefore, elements of plasma exert a force on one another even at large distances. It is this long-ranged Coulomb force that gives the plasma a large repertoire of possible motions and enriches the field of study known as plasma physics. In fact, the most interesting results concern so-called "collisionless" plasmas, in which the long-range electromagnetic forces are so much larger than the forces due to ordinary local collisions that the latter can be neglected altogether. By "collective behavior" we mean motions that depend not only on local conditions but on the state of the plasma in remote regions as well.

The word "plasma" seems to be a misnomer. It comes from the Greek πλάσμα, -ατος, τό, which means something molded or fabricated. Because of collective behavior, a plasma does not tend to conform to external influences; rather, it often behaves as if it had a mind of its own.

1.3 CONCEPT OF TEMPERATURE

Before proceeding further, it is well to review and extend our physical notions of "temperature." A gas in thermal equilibrium has particles of all velocities, and the most probable distribution of these velocities is known as the Maxwellian distribution. For simplicity, consider a gas in which the particles can move only in one dimension. (This is not entirely frivolous; a strong magnetic field, for instance, can constrain electrons to move only along the field lines.) The one-dimensional Maxwellian distribution is given by

$$f(u) = A \exp(-\tfrac{1}{2}mu^2/KT) \qquad [1\text{-}2]$$

where $f\,du$ is the number of particles per m^3 with velocity between u and $u + du$, $\tfrac{1}{2}mu^2$ is the kinetic energy, and K is Boltzmann's constant,

$$K = 1.38 \times 10^{-23}\ \mathrm{J/^\circ K}$$

The density n, or number of particles per m^3, is given by (see Fig. 1-2)

$$n = \int_{-\infty}^{\infty} f(u)\, du \qquad [1\text{-}3]$$

The constant A is related to the density n by (see Problem 1-2)

$$A = n\left(\frac{m}{2\pi KT}\right)^{1/2} \qquad [1\text{-}4]$$

The width of the distribution is characterized by the constant T, which we call the temperature. To see the exact meaning of T, we can

Adding this to $\mathbf{v}_R$, we have the total drift in a curved vacuum field:

$$\boxed{\mathbf{v}_R + \mathbf{v}_{\nabla B} = \frac{m}{q}\frac{\mathbf{R}_c \times \mathbf{B}}{R_c^2 B^2}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)} \quad (2\text{-}30)$$

It is unfortunate that these drifts add! This means that if one bends a magnetic field into a torus for the purpose of confining a thermonuclear plasma, the particles will drift out of the torus no matter how one juggles the temperatures and magnetic fields.

For a Maxwellian distribution, Eqs. [1-7] and [1-10] indicate that $\overline{v_\parallel^2}$ and $\frac{1}{2}\overline{v_\perp^2}$ are each equal to KT/m, since $v_\perp$ involves two degrees of freedom. Equations [2-3] and [1-6] then allow us to write the average curved-field drift as

$$\bar{\mathbf{v}}_{R+\nabla B} = \pm\frac{v_{th}^2}{R_c\,\omega_c}\hat{y} = \pm\frac{\bar{r}_L}{R_c}v_{th}\hat{y} \quad (2\text{-}30a)$$

where $\hat{y}$ here is the direction of $\mathbf{R}_c \times \mathbf{B}$. This shows that $\bar{\mathbf{v}}_{R+\nabla B}$ depends on the charge of the species but not on its mass.

2.3.3 $\nabla B \parallel \mathbf{B}$: Magnetic Mirrors

Now we consider a magnetic field which is pointed primarily in the z direction and whose magnitude varies in the z direction. Let the field be axisymmetric, with $B_\theta = 0$ and $\partial/\partial\theta = 0$. Since the lines of force converge and diverge, there is necessarily a component B_r (Fig. 2-7). We wish to show that this gives rise to a force which can trap a particle in a magnetic field.

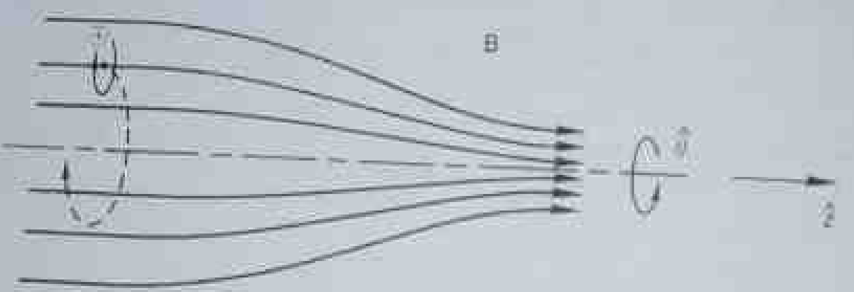

Fig. 2-7 Drift of a particle in a magnetic mirror field.

We can obtain B_r from $\nabla \cdot \mathbf{B} = 0$:

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0 \qquad [2\text{-}31]$$

If $\partial B_z/\partial z$ is given at $r = 0$ and does not vary much with r, we have approximately

$$rB_r = -\int_0^r r\frac{\partial B_z}{\partial z}\,dr \simeq -\frac{1}{2}r^2\left[\frac{\partial B_z}{\partial z}\right]_{r=0}$$

$$B_r = -\frac{1}{2}r\left[\frac{\partial B_z}{\partial z}\right]_{r=0} \qquad [2\text{-}32]$$

The variation of $|B|$ with r causes a grad-B drift of guiding centers about the axis of symmetry, but there is no radial grad-B drift, because $\partial B/\partial \theta = 0$. The components of the Lorentz force are

$$\begin{aligned} F_r &= q(\underset{1}{v_\theta B_z} - v_z B_\theta) \\ F_\theta &= q(\underset{2}{-v_r B_z} + \underset{3}{v_z B_r}) \\ F_z &= q(v_r B_\theta - \underset{4}{v_\theta B_r}) \end{aligned} \qquad [2\text{-}33]$$

Two terms vanish if $B_\theta = 0$, and terms 1 and 2 give rise to the usual Larmor gyration. Term 3 vanishes on the axis; when it does not vanish, this azimuthal force causes a drift in the radial direction. This drift merely makes the guiding centers follow the lines of force. Term 4 is the one we are interested in. Using Eq. [2-32], we obtain

$$F_z = \tfrac{1}{2}qv_\theta r(\partial B_z/\partial z) \qquad [2\text{-}34]$$

We must now average over one gyration. For simplicity, consider a particle whose guiding center lies on the axis. Then v_θ is a constant during a gyration; depending on the sign of q, v_θ is $\mp v_\perp$. Since $r = r_L$, the average force is

$$\bar{F}_z = \mp\frac{1}{2}qv_\perp r_L\frac{\partial B_z}{\partial z} = \mp\frac{1}{2}q\frac{v_\perp^2}{\omega_c}\frac{\partial B_z}{\partial z} = -\frac{1}{2}\frac{mv_\perp^2}{B}\frac{\partial B_z}{\partial z} \qquad [2\text{-}35]$$

We define the *magnetic moment* of the gyrating particle to be

$$\boxed{\mu \equiv \tfrac{1}{2}mv_\perp^2/B} \qquad [2\text{-}36]$$

so that

$$F_z = -\mu(\partial B_z/\partial z) \qquad [2\text{-}37]$$

This is a specific example of the force on a diamagnetic particle, which in general can be written

$$\mathbf{F}_\parallel = -\mu\,\partial B/\partial \mathbf{s} = -\mu\,\nabla_\parallel B \qquad [2\text{-}38]$$

where ds is a line element along $\mathbf{B}$. Note that the definition [2-36] is the same as the usual definition for the magnetic moment of a current loop with area A and current I: $\mu = IA$. In the case of a singly charged ion, I is generated by a charge e coming around $\omega_c/2\pi$ times a second: $I = e\omega_c/2\pi$. The area A is $\pi r_L^2 = \pi v_\perp^2/\omega_c^2$. Thus

$$\mu = \frac{\pi v_\perp^2}{\omega_c^2}\frac{e\omega_c}{2\pi} = \frac{1}{2}\frac{v_\perp^2 e}{\omega_c} = \frac{1}{2}\frac{mv_\perp^2}{B}$$

As the particle moves into regions of stronger or weaker $\mathbf{B}$, its Larmor radius changes, but μ *remains invariant*. To prove this, consider the component of the equation of motion along $\mathbf{B}$:

$$m\frac{dv_\parallel}{dt} = -\mu\frac{\partial B}{\partial s} \qquad [2\text{-}39]$$

Multiplying by $v_\parallel$ on the left and its equivalent ds/dt on the right, we have

$$mv_\parallel\frac{dv_\parallel}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{\partial B}{\partial s}\frac{ds}{dt} = -\mu\frac{dB}{dt} \qquad [2\text{-}40]$$

Here dB/dt is the variation of B as seen by the particle; B itself is constant. The particle's energy must be conserved, so we have

$$\frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2\right) = \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2 + \mu B\right) = 0 \qquad [2\text{-}41]$$

With Eq. [2-40] this becomes

$$-\mu\frac{dB}{dt} + \frac{d}{dt}(\mu B) = 0$$

so that

$$d\mu/dt = 0 \qquad [2\text{-}42]$$

The invariance of μ is the basis for one of the primary schemes for plasma confinement: the *magnetic mirror*. As a particle moves from a weak-field region to a strong-field region in the course of its thermal

A plasma trapped between magnetic mirrors. FIGURE 2-8

motion, it sees an increasing B, and therefore its $v_\perp$ must increase in order to keep μ constant. Since its total energy must remain constant, $v_\parallel$ must necessarily decrease. If B is high enough in the "throat" of the mirror, $v_\parallel$ eventually becomes zero; and the particle is "reflected" back to the weak-field region. It is, of course, the force $\mathbf{F}_\parallel$ which causes the reflection. The nonuniform field of a simple pair of coils forms two magnetic mirrors between which a plasma can be trapped (Fig. 2-8). This effect works on both ions and electrons.

The trapping is not perfect, however. For instance, a particle with $v_\perp = 0$ will have no magnetic moment and will not feel any force along $\mathbf{B}$. A particle with small $v_\perp/v_\parallel$ at the midplane ($B = B_0$) will also escape if the maximum field B_m is not large enough. For given B_0 and B_m, which particles will escape? A particle with $v_\perp = v_{\perp 0}$ and $v_\parallel = v_{\parallel 0}$ at the midplane will have $v_\perp = v_\perp'$ and $v_\parallel = 0$ at its turning point. Let the field be B' there. Then the invariance of μ yields

$$\tfrac{1}{2}mv_{\perp 0}^2/B_0 = \tfrac{1}{2}mv_\perp'^2/B' \qquad [2\text{-}43]$$

Conservation of energy requires

$$v_\perp'^2 = v_{\perp 0}^2 + v_{\parallel 0}^2 \equiv v_0^2 \qquad [2\text{-}44]$$

Combining Eqs. [2-43] and [2-44], we find

$$\frac{B_0}{B'} = \frac{v_{\perp 0}^2}{v_\perp'^2} = \frac{v_{\perp 0}^2}{v_0^2} \equiv \sin^2\theta \qquad [2\text{-}45]$$

where θ is the pitch angle of the orbit in the weak-field region. Particles with smaller θ will mirror in regions of higher B. If θ is too small, B' exceeds B_m, and the particle does not mirror at all. Replacing B' by B_m in Eq. [2-45], we see that the smallest θ of a confined particle is given by

$$\sin^2\theta_m = B_0/B_m \equiv 1/R_m \qquad [2\text{-}46]$$

Magnetic Mirror $\nabla B \parallel B$

Now we consider a magnetic field which is pointed in the z-direction & whose magnitude varies in the z-direction. Let the field lines are symmetric along z-axis & B is not vary with the θ-direction. So,

$$\frac{\partial B_\theta}{\partial \theta} = 0 \quad , \quad B_\theta = 0$$

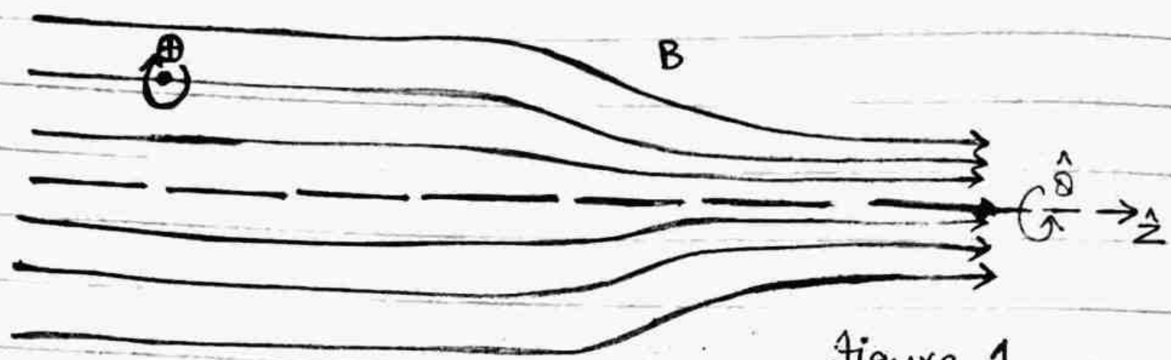

Figure 1

Since the lines of force converge or diverge, there is necessarily a component B as shown in Figure 1. We show that this give rise to a force which can trap or confined a particle in a magnetic field.

In our case,

By Maxwell's Eq.

$$\nabla \cdot B = 0$$

Expression of $\nabla \cdot B$ in spherical coordinates

$$\nabla \cdot B = \frac{1}{r}\left(r \frac{\partial B_r}{\partial r}\right) + \frac{1}{r} \frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0$$

$$\frac{1}{r} \frac{\partial B_\theta}{\partial \theta} = 0$$

So,

$$\frac{1}{r}\left(\frac{r\,\partial B_r}{\partial r}\right) = -\frac{\partial B_z}{\partial z}$$

$$\frac{r\,\partial B_r}{\partial r} = -r\frac{\partial B_z}{\partial z}$$

$$r\,\partial B_r = -r\frac{\partial B_z}{\partial z}\,\partial r \quad \text{——} \; (1)$$

Integrating eq. (1),

$$\int_0^r r\,\partial B_r = \int_0^r -r\frac{\partial B_z}{\partial z}\,\partial r$$

$$\cancel{r} B_r = -\frac{r^{\cancel{2}}}{2}\frac{\partial B_z}{\partial z}$$

$$B_r = \frac{-r}{2}\frac{\partial B_z}{\partial z} \quad \text{——} \; (2)$$

Now,

By Lorentz Eq.

$$f = q(V\times B)$$

We don't need electric field in this case, so ignoring it,

$$V\times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_r & V_\theta & V_z \\ B_r & B_\theta & B_z \end{vmatrix}$$

r- component :-

$$f_r = q(V_\theta B_z - V_z B_\theta) \quad \text{——} \; (3)$$

θ- component :-

$$f_\theta = -q(V_r B_z - V_z B_r)$$

$$f_\theta = q(V_z B_r - V_r B_z) \quad \text{——} \quad (4)$$

z- component :-

$$f_z = q(V_r B_\theta - V_\theta B_r) \quad \text{——} \quad (5)$$

as $B_\theta = 0$

So, $V_z B_\theta$ in eq. (3) & $V_r B_\theta$ in eq. (5) equals to zero.

So,

$$f_r = q(V_\theta B_z)$$

$$f_\theta = q(V_z B_r - V_r B_z)$$

$$f_z = q(-V_\theta B_r)$$

The term $V_\theta B_z$ & $V_z B_r$ has the usual Larmor gyration & $V_r B_z$ give drift in radial direction which is in our case not so important. So, it means this term has no concerned with confinement.

So, we are left with, $-V_\theta B_r$ which gives the drift in a parallel direction & in our case this term is imp

$$f_z = -q V_\theta B_r \quad \text{——} \quad (6)$$

put value of B_r from eq. (2) in eq. (6),

$$f_z = qV\theta \frac{r}{2} \frac{\partial B_z}{\partial z}$$

$$f_z = qV_\perp \frac{r}{2} \frac{\partial B_z}{\partial z}$$ $\because \theta$ is $\perp r$

put value of radius, $\because r_L = \frac{V_\perp}{\omega_c}$

$$f_z = \pm qV_\perp \frac{V_\perp}{2\omega_c} \frac{\partial B_z}{\partial z}$$

put value of ω, $\because \omega = \frac{qB}{m}$

$$f_z = -q \frac{V_\perp^2}{\frac{2qB}{m}} \frac{\partial B_z}{\partial z}$$

$$f_z = -\frac{mV_\perp^2}{2B} \frac{\partial B_z}{\partial z}$$

for restoring force

$$f_z = \frac{-1}{2} \frac{mV_\perp^2}{B} \frac{\partial B_z}{\partial z}$$

$\because$ we use -ive sign here only to trap particle in a magnetic field.

This quantity is known as magnetic moment because it has the dimensions of magnetic moment.

$$\mu = \frac{1}{2} \frac{mV_\perp^2}{B}$$

So,

$$f_z = -\mu \frac{\partial B_z}{\partial z}$$

$$f_z = -\mu \nabla_{\parallel} B \quad \text{—— (7)}$$

In general,

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$$f_s = -\mu \frac{\partial B}{\partial s}$$

where s is the displacement.

In general, magnetic moment $= \mu = IA$ — (8)

where A is the area element $= \pi r^2$
$= \pi r_\ell^2$

& $I = \frac{q}{t}$

put in eq. (8).

$$\mu = \frac{q}{t} \pi r_\ell^2 \quad \text{— (9)}$$

where t is the time to complete 1 gyration.

So, $t = \frac{2\pi}{\omega_c}$ also, $r_\ell = \frac{V_\perp}{\omega_c}$

where put values in eq. (9),

$$\mu = \frac{q}{\frac{2\pi}{\omega_c}} \frac{\pi V_\perp^2}{\omega_c^2}$$

$$\mu = \frac{q}{2} \frac{V_\perp^2}{\omega_c}$$

put value of ω

$$\mu = \frac{q}{2} \frac{V_\perp^2}{qB/m}$$

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$$\mu = \frac{1}{2}\frac{mV_\perp^2}{B}$$

f_s is a parallel force so, it can be written as,

$$ma = -\mu\frac{\partial B}{\partial s}$$

$$m\frac{dV_{\parallel}}{dt} = -\mu\frac{\partial B}{\partial s} \quad \text{---} \ (10)$$

Multiplying b/s of eq. (10) by $V_{\parallel}$,

$$mV_{\parallel}\frac{dV_{\parallel}}{dt} = -\mu\frac{\partial B}{\partial s}\cdot V_{\parallel}$$

$$\frac{d}{dt}\left(\frac{1}{2}mV_{\parallel}^2\right) = -\mu\frac{\partial B}{\partial s}\cdot\frac{ds}{dt}$$

$$= -\mu\frac{\partial B}{\partial t}$$

$\because \frac{d}{dt}\left(\frac{1}{2}mV_{\parallel}^2\right)$

$\cancel{2}\frac{1}{\cancel{2}}mV_{\parallel}\frac{dV_{\parallel}}{dt}$

$mV_{\parallel}\frac{dV_{\parallel}}{dt}$

By law of conservation of Energy,

$$\frac{d}{dt}\left(\frac{1}{2}mV_{\parallel}^2 + \frac{1}{2}mV_\perp^2\right) = 0$$

$$\frac{d}{dt}\frac{1}{2}mV_{\parallel}^2 + \frac{d}{dt}\frac{1}{2}mV_\perp^2 = 0$$

By putting values,

$$-\mu\frac{\partial B}{\partial t} + \frac{d}{dt}\frac{1}{2}mV_\perp^2 = 0 \quad \text{---} \ (11)$$

From the value of μ,

$$\mu = \frac{1}{2}\frac{mV_\perp^2}{B}$$

$$\mu B = \frac{1}{2}mV_\perp^2$$

putting values in eq. (11),

$$-\mu\frac{dB}{dt} + \frac{d}{dt}(\mu B) = 0$$

$$-\cancel{\mu\frac{dB}{dt}} + \cancel{\mu\frac{dB}{dt}} + B\frac{d\mu}{dt} = 0$$

$$B\frac{d\mu}{dt} = 0$$

here, $B \neq 0$

It means,

$$\frac{d\mu}{dt} = 0$$

So, μ is constant.

$$\mu = \text{constant}$$

$$C = \frac{1}{2}\frac{mV_\perp^2}{B}$$

Now,

If particle move in strong magnetic field it means B is increasing so, $V_\perp$ also increases then, from law of conservation of energy $V_\parallel$ should decrease. So, particle donot move in parallel direction. Now, particle move only in the $\perp$r direction. Same process is repeated on the other side, in this way particle is confined in the magnetic mirror machine. This effect works on both ions & electrons.