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As know

$$\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\text{grad} \frac{1}{|\vec{r}-\vec{r}'|}$$

Put this in (A)

$$\vec{A}(x,y,z) = \frac{-\mu_0}{4\pi}\int_v \vec{M}(x',y',z') \times \text{grad} \frac{1}{|\vec{r}-\vec{r}'|} dv' \rightarrow (B)$$

By using vector notation or vector identity

$$\therefore \text{curl } UV = U \text{ curl} V + \underset{\text{grad}}{\nabla U} \times V$$

Let $\vec{M} = V$; $\frac{1}{|\vec{r}-\vec{r}'|} = U$

$$\text{curl}'\left(\frac{\vec{M}}{|\vec{r}-\vec{r}'|}\right) = \frac{1}{|\vec{r}-\vec{r}'|}\text{curl}\vec{M} + \text{grad}\frac{1}{|\vec{r}-\vec{r}'|} \times \vec{M}$$

Put this value in (B)

$$\vec{A}(x,y,z) = -\frac{\mu_0}{4\pi}\int_{V_0} \text{curl}'\left(\frac{\vec{M}}{|\vec{r}-\vec{r}'|}\right)$$

$$\vec{A}(x,y,z) = -\frac{\mu_0}{4\pi}\int_{V_0}\left[\frac{1}{|\vec{r}-\vec{r}'|}\text{curl}'\vec{M} + \text{grad}\frac{1}{|\vec{r}-\vec{r}'|} \times \vec{M}\right]$$

$$\vec{A}(x,y,z) = \frac{\mu_0}{4\pi}\int_{V_0}\frac{\text{curl}\vec{M}\,dv'}{|\vec{r}-\vec{r}'|} - \frac{\mu_0}{4\pi}\int_{V_0}\text{curl}'\left(\frac{M}{|\vec{r}-\vec{r}'|}\right)dv' \rightarrow (C)$$

Now By using Stoke's Theorem

$$\int \text{curl}\,\vec{F}\cdot dv = \int_S \hat{n}\times\vec{F}\,da$$

The second term of Eq (C) becomes

$$\vec{A}(x,y,z) = \frac{\mu_0}{4\pi}\int_{V_0}\frac{\text{curl}\,\vec{M}}{|\vec{r}-\vec{r}'|} - \frac{\mu_0}{4\pi}\int_S \frac{\hat{n}\times\vec{M}}{|\vec{r}-\vec{r}'|}\,da'$$

Now By Inter Changing

$$\hat{n}\times\vec{M} = -\vec{M}\times\hat{n}$$

$$\vec{A}(x,y,z) = \frac{\mu_0}{4\pi}\int_{V_0}\frac{\text{curl}\,\vec{M}}{|\vec{r}-\vec{r}'|} + \frac{\mu_0}{4\pi}\int_S \frac{\vec{M}\times\hat{n}}{|\vec{r}-\vec{r}'|}\,da' \longrightarrow (D)$$

curl $\vec{M}$ is the current density $\vec{J_m}$ due to magnetization. where S_0 is the surface of V_0. ~~jm is current density over the surface~~

so,

$$\vec{J_m} = \text{curl}\,\vec{M} \longrightarrow (i)$$

$$\vec{j_m} = \vec{M}\times\hat{n} \longrightarrow (ii)$$

put this in Eq (D) so Eq becomes

$$\vec{A}(r) = \vec{A}(x,y,z) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J_m}}{|\vec{r}-\vec{r}'|}\,dv' + \frac{\mu_0}{4\pi}\int_S \frac{\vec{j_m}}{|\vec{r}-\vec{r}'|}\,da \longrightarrow (1)$$

where

j_m is the magnetization current per unit lenght flowing in the surface. $\vec{J_m}$ is the magnetization current density.

The vector potential produced by a distribution of atomic currents inside matter has the

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same form as that produced by a distribution of true transport currents Eq(4) is the proper expression for the surface current density Abrupt change is occure due to discontinuity surface current density which is consistent with

$$\vec{J_m} = \text{curl}\,\vec{M}$$

$\vec{J_m}$ must be introduced wherever $\vec{M}$ changes abruptly, as it might at the interface between two medium but if the region of discontinuity in $\vec{M}$ is imagined to be splend out over the distance $\Delta\xi$ then it can be shown that $\vec{J_m}$ is contained in the term

$$\bar{J}_m \,\Delta\xi$$

This Equation present some practical difficulties when it comes to the computing $\vec{B}$ from a specified distribution of magnetization Gradient of a scalar field is easier to compute than the curl of a vector field

Again we take

$$\vec{B}(r) = \text{curl}\,\vec{A} = \frac{\mu_0}{4\pi}\int_v \frac{\text{curl}\,\vec{M} \times (\vec{r}-\vec{r}')\,dv'}{|\vec{r}-\vec{r}'|} \quad \rightarrow (2)$$

Again Using vector Notation As,

$$\text{curl}(U \times V) = \vec{U}\,\text{div}\,\vec{V} - \vec{V}\,\text{div}\,\vec{U} + (V.\vec{\nabla})\vec{U} - (\vec{U}.\vec{\nabla})\vec{V}$$

In our case

$$U = \vec{M} \qquad V = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

So,

$$\text{curl}\left[\vec{M} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right] = \vec{M}\,\text{div}\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right) - \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)\text{div}\,\vec{M}$$

$$+\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}.\nabla\right)\vec{M} - \left(\vec{M}.\vec{\nabla}\right)\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

Since $\text{div}\,\vec{M}(x', y', z') = 0$, There fore Second and third term become Zero. The Remaining Eq When put in Eq (2) is

$$\vec{B}(r) = \frac{\mu_0}{4\pi}\left[\int_{V_0} \vec{M}\,\text{div}\,\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,dv' - \int_{V_0} (\vec{M}.\vec{\nabla})\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,dv'\right]$$

$$\Rightarrow \vec{B}(r) = \vec{B_I}(r) + \vec{B_{II}}(r)$$

Now

$$B_I(r) = \frac{\mu_0}{4\pi}\int_{V_0} \vec{M}\,\text{div}\,\frac{r - r'}{|\vec{r}-\vec{r}'|^3}\,dv'$$

Changing volume Integral into

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Surface Integral

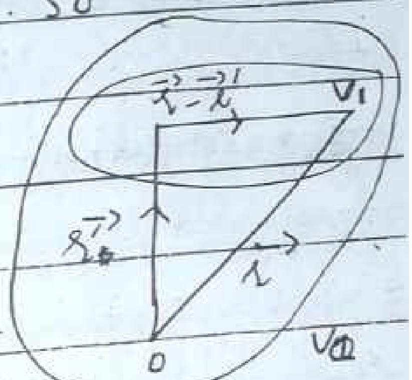
$$\vec{B}_1(r) = \frac{\mu_0}{4\pi}\int_S \frac{\vec{M}\,(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\cdot\hat{n}\,da'$$

$$\vec{B}_1(r) = \frac{\mu_0 \vec{M}}{4\pi}\int_S \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\,\hat{n}\,da$$

It is a solid angle. So

$$\vec{B}_1(r) = \frac{\mu_0 \vec{M} \times \cancel{4\pi}}{\cancel{4\pi}}$$

$$\boxed{\vec{B}_1(r) = \mu_0\vec{M}}$$



In above we consider first $\vec{B}_1$ integral. It is convinient to divide the volume of the magnet, V_0 into two region ① a small spherical region V_1 surrounding the point (x,y,z) and ② for remaining volum $V_0 - V_1$. Since the divergence turn into integral vanishes every where except at the point $\vec{r} = \vec{r}\,'$. The integral over $V_0 - V_1$ is zero. Now we consider B_{11} integral. Then the integral may be transformed by mean of a second identity i-e

$$\vec{B}_{11}(r) = -\frac{\mu_0}{4\pi}\int_{V'} (\vec{M}\cdot\vec{\nabla})\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}\,dv$$

Now Apply vector notation.

$$\nabla\left[\vec{M}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right] = \vec{M}\cdot\nabla\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} + M\ \text{curl}\left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)$$

here $\text{curl}\,\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\text{curl grad}\,\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = 0$

So

$$\vec{B}_{II}(r) = -\frac{\mu_0}{4\pi}\int_{v'}\vec{\nabla}\left(\vec{M}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\right)dv'$$

$$\vec{B}_{II}(r) = -\frac{\mu_0}{4\pi}\vec{\nabla}\int_{v'}\vec{M}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,dv'$$

$$\vec{B}_{II}(r) = -\mu_0\vec{\nabla}\cdot\frac{1}{4\pi}\int_{v'}\vec{M}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,dv'$$

$$\vec{B}_{II}(r) = -\mu_0\vec{\nabla}U^*(r)$$

The Quantity $U^*(r)$ is the scalar field we shall call it the magnetic scalar potential due to magnetic material.

$$U^*(r) = \frac{1}{4\pi}\int_{v'}\vec{M}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,dv'$$

So,

$$\vec{B}(r) = \vec{B}_I(r) + \vec{B}_{II}(r)$$

$$\vec{B}(r) = \mu_0\vec{M} - \mu_0\vec{\nabla}U^*(r)$$

The first term is the local magnetiz while second term is grad of scalar potential.

Thus the Magnetic Induction due to a magnetized distribution of matter may be expressed as the sum of two terms in the gradient of the scalar field and a term proportional to the local magnetization.

At an External point i-e in vaccume $\vec{M}$ is zero. M.I is Then just the gradient of the scalar field. Which is the integral of the distant dipole field.

Magnetic Scalar Potential And Magnetic Pole Density:-

As magnetic scalar Potential is written as

$$U^*(r) = \frac{1}{4\pi} \int_{V_0} \frac{\vec{M}(r') \cdot (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

It is similar form to that for the Electrostatic potential arising from polarized dielectric material. Now we apply the vector Identity

$$U^*(r) = \frac{1}{4\pi} \int_{V_0} \vec{M}(r) \cdot \text{grad} \frac{1}{|\vec{r} - \vec{r}'|} dv'$$

we ignore (-) sign. Now we take dot product of

$$\frac{\vec{M}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \vec{M}\,\text{grad}\,\frac{1}{|\vec{r}-\vec{r}'|} = \frac{\text{div}\,\vec{M}}{|\vec{r}-\vec{r}'|} - \frac{1}{|\vec{r}-\vec{r}'|}\,\text{div}\vec{M}$$

So by putting we have.

$$U^*(r) = \frac{1}{4\pi}\int_{V_0}\left[\frac{\text{div}\,M}{|\vec{r}-\vec{r}'|}\,dv' - \frac{1}{|\vec{r}-\vec{r}'|}\,\text{div}'\vec{M}\,dv'\right]$$

$$U^*(r) = \frac{1}{4\pi}\int_{V_0}\frac{\vec{M}\,\text{div}\,\vec{M}}{|\vec{r}-\vec{r}'|}\,dv' - \frac{1}{4\pi}\int_{V_0}\frac{1}{|\vec{r}-\vec{r}'|}\,\text{div}'\vec{M}\,dv'$$

Taking 1st integral as a surface integral

$$U^*(r) = \frac{1}{4\pi}\int_{S_0}\frac{\vec{M}\cdot\hat{n}}{|\vec{r}-\vec{r}'|}\,d\vec{r} - \frac{1}{4\pi}\int_{V_0}\frac{1}{|\vec{r}-\vec{r}'|}\,\text{div}'\vec{M}\,dv'$$

1st integral shows surface charge density represented by σ. Where S_0 is the surface of the region V_0. We include $\vec{M}$ to show magnetic field. Also the 2nd integral Equal to volume charge density. For this we defined two scalar quantities

$$\rho(m)r' = -\,\text{div}\,\vec{M}(r')$$

Known as magnetic pole density. And

$$\sigma m(r') = \vec{M}\cdot\hat{n}$$

the surface density of Magnetic pole strenght. If there is uniform magnetized bar then the 2nd integral we vanish

$$U^*(r) = \frac{1}{4\pi}\int_{S_0}\frac{\sigma_m(r')\,da'}{|\vec{r}-\vec{r}'|} + \frac{1}{4\pi}\int_{V_0}\frac{\rho_m(r')\,dv'}{|\vec{r}-\vec{r}'|}$$

because

$$\rho_m = 0$$

So

$$U^*(r) = \frac{1}{4\pi}\int_{S_0}\frac{\sigma_m(r')\,da'}{|\vec{r}-\vec{r}'|}$$

Normal component of magnetized material is left. This statement relates to divergence Theorem

$$\int_S \vec{m}\cdot\hat{n}\,da + \int_{V_0}(-\operatorname{div}\vec{M})\,dv' = 0$$

The total Pole strenght of Every magnet is zero. i-e

$$\nabla\cdot B = 0$$

Now we apply divergence theorem to convert ~~volume~~ integral to volume surface integral. Hence $\vec{B}(x,y,z)$ is Obtained as $-\mu_0$ times the gradient w.r.t the unprimed coordinates.

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$$\vec{B} = -\mu_0 \nabla U^*(r) + \mu_0 \vec{M}(r)$$

$$\vec{B}(= -\frac{\mu_0}{4\pi} \nabla \int_{S_0} \frac{\sigma_m(r')}{|\vec{r}-\vec{r}'|} da' - \frac{\mu_0}{4\pi} \nabla \int_{V_0} \frac{\rho_m(r')}{|\vec{r}-\vec{r}'|} dv'$$

$$+ \mu_0 \vec{M}(r)$$

This Eq shows M.I at any far point.

$$\vec{B} = +\frac{\mu_0}{4\pi} \int_{S_0} \frac{\sigma_m(r')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} da' + \frac{\mu_0}{4\pi} \int_{V_0} \frac{\rho_m(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|} dv'$$

$$+ \mu_0 \vec{M}(r)$$

This Eq represents the contribution from the magnetized material in V_0 to magnetic Induction at (x, y, z).

Sources Of the Magnetic Field:-

⇒ (1)

Magnetic Intensity:-

It is important to know that under certain conditions the same piece of matter produces magnetic field both because of magnetization and also it is carrying a current of charge carriers. The best magnetic materials are Fe, Co, Ni etc. They carry curren

because of free Electrons
one of our best magnetic material iron may carry a current due to free Electrons but fixed iron ions in the crystal contain atomic current, which can be oriented to produce a strong magnetization.
The General Expression for Magnetic field is

$$\vec{B}(r) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(r)\,(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|}\,dv' - \mu_0 \nabla U^*(r) + \mu_0 \vec{M}(r) \longrightarrow (1)$$

where

$$U^*(r) = \frac{1}{4\pi}\int_V \frac{\rho_m}{|\vec{r}-\vec{r}'|}\,dv' + \frac{1}{4\pi}\int_S \frac{\sigma_m\,da'}{|\vec{r}-\vec{r}'|}$$

The volume V extends over all current carrying region and over all matter, the surface S includes all surfaces and interfaces between different media.
The current density $\vec{J}$ includes all currents of the charge transport whereas the Effect of atomic currents is found in the magnetization vector $\vec{M}$.
Eq (1) may be solved for $\vec{B}$ if $\vec{M}$ and

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$\vec{r}$ are specified at all points. $\vec{M}(x', y', z')$ depends on $\vec{B}(x', y', z')$ so that even if the functional form of $\vec{M}(\vec{B})$ is known the eq (1) provides at best an integral Eq for $\vec{B}$. We add contribution because of current density as current is flowing through atoms.

As,
$$\vec{B} = \mu_0 \vec{H}$$
$$\Rightarrow \vec{H} = \frac{\vec{B}}{\mu_0}$$

The magnetic material also has magnetization. The magnetic Intensity $\vec{H}$ defined by
$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Putting value of $\vec{B}$ in above.
$$\vec{H} = \frac{\cancel{\mu_0}}{\cancel{\mu_0} 4\pi} \int_v \frac{\vec{J} \times (\vec{r} - \vec{r}\,')\, dv'}{|\vec{r} - \vec{r}\,'|} - \nabla U^*(r) + M(r) - M$$

$$\vec{H} = \frac{1}{4\pi} \int_v \frac{\vec{J} \times (\vec{r} - \vec{r}\,')\, dv'}{|\vec{r} - \vec{r}\,'|} - \nabla U^*(r) + M(r) - M$$

It appears that we have gained nothing by this work because $\vec{H}$ still depends on $\vec{M}$ through ρ_m & σ_m. A Magnetic material which produces magnetization by it self and a non-magnetic material, which induces magnetization by near by object has same result. But now we can show how $\vec{H}$ related to the current density $\vec{J}$ through a differential Equation.

The Field Equation:-

The basic Equations describing the magnetic Effects of conventional currents were Expressed in differential form is

$$\mathrm{div}\,\vec{B} = 0$$

$$\mathrm{curl}\,\vec{B} = \mu_0 J$$

we should now like to see how these Equations are modified when the Magnetic field $\vec{B}$ includes a contribution from magnetized material. The divergence Equation came about because $\vec{B}$ could be written as the curl of the vector function $\vec{A}$. But this result is not limited to magnetic fields produced by conventional currents. The field produced by magnetized matter is also derivable from a vector potential.

Thus $\vec{B}$ may always be written as curl $\vec{A}$ and the divergence Equation follows.

$$\operatorname{div} \vec{B} = 0 \longrightarrow (1)$$

The "curl Equation" is the differential form of Ampere's circuital law. Due to magnetization the Eq becomes.

$$\operatorname{curl} \vec{B} = \mu_0 (J + J_m)$$

where $\vec{J_m}$ is the current density due to magnetization. As,

$$\operatorname{curl} \vec{B} = \mu_0 \vec{\vec{J}} = \mu_0 (\vec{\vec{J}} + \vec{\vec{J}}_m) \longrightarrow (2)$$

$$\vec{J_m} = \operatorname{curl} \vec{M}$$

Put this value in Eq (2)

$$\operatorname{curl} \vec{B} = \mu_0 (\vec{\vec{J}} + \operatorname{curl} \vec{M})$$

$$\frac{\operatorname{curl} \vec{B}}{\mu_0} = \vec{J} + \operatorname{curl} \vec{M} \longrightarrow (3)$$

$$\cancel{\operatorname{curl} \frac{\vec{B}}{\mu_0} - \operatorname{curl} \vec{M} = \vec{J}}$$

$$\cancel{\vec{J} = \operatorname{curl} \left[\frac{\vec{B}}{\mu_0} - \vec{M} \right]}$$

$$\operatorname{curl} \vec{H} = \vec{J} + \operatorname{curl} \vec{M}$$

$\therefore B = \mu$

As

$H = $

$$B = \mu_0 H$$

Due to magnetization

$$B = \mu_0 [\vec{H} + \vec{M}]$$

$$\frac{\vec{B}}{\mu_0} = \vec{H} + \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

So Equation (3) written as

$$\text{curl } \frac{\vec{B}}{\mu_0} - \text{Curl } \vec{M} = \vec{J}$$

$$\text{curl} \left[\frac{\vec{B}}{\mu_0} - \vec{M} \right] = \vec{J}$$

$$\text{curl } \vec{H} = \vec{J} \longrightarrow (4)$$

Hence the auxilliary Magnetic vector $\vec{H}$ is related to the current density through its curl Eq (1) and (4) are fundamental magnetic field Equation.

These Equations together with appropriate boundary conditions and an Experimental relation ship between $\vec{B}$ and $\vec{H}$ are sufficient to solve Magnetic problem.

Magnetic field $\vec{H}$ relates with only current density. Now take the surface integral of Eq (4)

$$\int_S \text{curl } \vec{H} \cdot \hat{n}\, dn = \int_S \vec{J} \cdot \hat{n}\, dn$$

Now Apply divergence Theorem to convert surface Integral in to line integral.

$$\oint_c \vec{H} \cdot dl = \int_S \vec{J} \cdot \hat{n}\, da$$

So,

$$\oint_c \vec{H} \cdot dl = I$$ "current flows on surface"

The line integral of $\vec{H}$ is Equal to current.

Now take volume integral of

$\nabla \cdot \vec{B}$

For more simplification

$$\int_c \operatorname{div} \vec{B}\, dv = 0$$

The line integral of the tangential component of the magnetic intensity around a closed path C is Equal to the Entire transport current through the area bounded by the curve C.

By using divergence Theorem.

$$\oint_S \vec{B} \cdot \hat{n}\, da = 0$$

The Magnetic flux through any closed surface is zero.