$$B(r_2) = \frac{\mu_0}{4\pi}\left[\frac{-m_x}{r_2^3} + \frac{3\, m_x\, r_2 \cdot r_2}{r_2^{-5}}\right] \quad 19$$ 37

This Equation shows the M.F of a distant circuit does not depend on its detailed geometry but only its Magnetic Moment m. This Eq of the same form as the E.F due to an Electric dipole which Explain the same Magnetic dipole field, $\vec{m}$ is usually called the "Magnetic dipole Moment of the circuit".

The Magnetic Scalar Potential :-

$\nabla \times B = \mu_0 J$ (4.41) ampere's law

The div B = 0 but curl $\vec{B} = \mu_0 \vec{J} = 0$ when current density is zero. Scalar is always included with gradiant. If we have a scalar U then the gradiant of a scalar potential is

Exchange in pot = current produced

$$\vec{B} = -\mu_0 \text{ grad } U^* \longrightarrow (I)$$

In Actual U is the P.E which is changing and is decreasing so '−' sign is included.

As the divergence of $\vec{B}$ is also zero so,

$$\text{div } \vec{B} = -\mu_0 (\text{div grad } U) = 0$$

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$$\operatorname{div} \vec{B} = -\mu_0 \nabla^2 U^* = 0$$

Where U^* is the magnetic scalar potential which satisfy the Laplace Equation.

Much of the work of Electrostatic can be taken over directly and used to Evaluate U^* for various situation however case must be taken in applying the boundary condition. The Expression for the scalar potential of a Magnetic dipole is particularly useful. If it is noted that the relation.

$$= \frac{-\mu_0}{4\pi}\left(\vec{m} \cdot \operatorname{grad} \frac{\vec{r_2}}{r_2^3}\right)$$

we have a vector notation of grad of two vectors

$$\operatorname{grad}\left(\vec{m} \cdot \frac{\vec{r_2}}{r_2^3}\right) = m \cdot \operatorname{grad} \frac{\vec{r_2}}{r_2^3} + \frac{\vec{r_2}}{r_2^3} \cdot \operatorname{grad} \vec{m}$$

$$+ m \operatorname{curl} \frac{\vec{r_2}}{r_2^3} + \frac{\vec{r_2}}{r_2^3} \cdot \operatorname{curl} \vec{m}$$

As $\operatorname{grad} \vec{m} = 0$ Also $\operatorname{curl} \vec{m} = 0$

So we left with Eq

$$\text{curl}\ \frac{\vec{r_2}}{r_2^3} = \nabla \frac{1}{r_2^3} \times \vec{r_2} + \frac{1}{r_2^3}\ \text{curl}\ \vec{r_2}$$

$$\Rightarrow B(r_1) = -\frac{\mu_0}{4\pi}\left(m.\,\text{grad}\ \frac{\vec{r_2}}{r_2^3} + m\ \text{curl}\ \frac{\vec{r_2}}{r_2^3}\right)$$

$$= -\frac{\mu_0}{4\pi}\left(m.\,\text{grad}\ \frac{r_2}{r_2^3} + m\left(\nabla \frac{r_2}{r_2^3}\right) + m\ \text{curl}\ \frac{r_2}{r_2^3}\right)$$

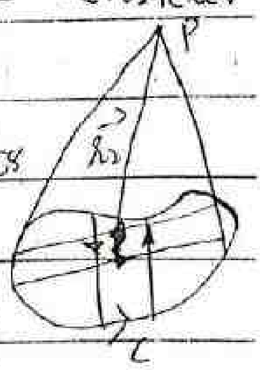
Hence

$$\vec{B} = -\frac{\mu_0}{4\pi}\ \text{grad}\left(\frac{\vec{m}\cdot\vec{r_2}}{r_2^3}\right) \longrightarrow \text{II}$$

By comparing Eq I & Eq II

$$U^* = \frac{1}{4\pi}\left(\frac{\vec{m}\cdot\vec{r_2}}{r_2^3}\right)$$

For a dipole Moment $\vec{m}$ we consider a large circuit 'C' can be divided into many small circuits as shown in above figure.



If each small loop formed by the mesh carries the same current as originally was carried by the circuit C, Then because of the cancellation of currents in the common branch of adjacent loops. The net effect is the same as if the charge flowed

in the circuit 'C'.
For any one of the small loops the magnetic moment may be written as

$$d\vec{m} = I\hat{n}\,da$$

Since each of the loops is sufficiently small to be regarded as planes using this expression and integrating over the surface bounded by 'C' gives

$$U^*(P) = \frac{I}{4\pi}\int_S \frac{\hat{n}\cdot\vec{r_2}}{r_2^3}\,da$$

In this equation $\vec{r_2}$ must be interpreted as the vector from 'da' to the point 'P'. That is $\vec{r_2}$ shown in figure.
Making the change $\vec{r_2} = -\vec{r}$ results as.

$$U^*(P) = \frac{I}{4\pi}\int_S \frac{\vec{r}\cdot\hat{n}}{r^3}\,da$$

The quantity $\vec{r_2}.\hat{n}\,da$ is just $\vec{r}$ times the projection of da on a plane perpendicular to $\vec{r_2}$.
Thus $\vec{r_2}.\hat{n}\,da/r^3$ is the solid angle substended by 'da' at 'P'. So eq may written as

$$U^*(P) = -\frac{I\,\Omega}{4\pi} \qquad \because \Omega = \frac{\vec{r}_{..}\cdot\hat{n}\,da}{r^3}$$

where Ω is the solid angle subtended by the curve 'C' at the point 'P'.
The Magnetic scalar potential can be used for the calculation of the Magnetic field due to Either current carrying circuits or to magnetic double layers (layers of dipole). This Procedure is occasionally useful in dealing with circuit problem; however its principle used in dealing with magnetic materials.

Chapter # 2
"Magnetic Properties Of Matter Magnetization"

Magnetic Moment:-

current flowing through any magnetic material is called as magnetic moment. Each atomic current is a tiny closed circuit of atomic dimension is appropriately described as a magnetic dipole moment.

Explanation:-

Let the magnetic moment of the i^{th} particle/atom be $\vec{m_i}$, Magnetization is defined as we sum up vectorially all of the dipole moments in a small volume Element ΔV, and then divide the result by ΔV. The resulting Quantity is then as.

$$\vec{M} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_i \vec{m_i}$$

From this we simply say that Magnetic moment actually measures atom current circuit per unit Volume. Also known as magnetization.

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Explanation:- (Remaining).
But in the presence of an External Exciting field $\vec{M}$, usually $\vec{m}$ depends upon this Field.
The vector function $\vec{M}$ provides the macroscopic discription of the atomic currents inside the matter.
Specifically $\vec{M}$, measures the number

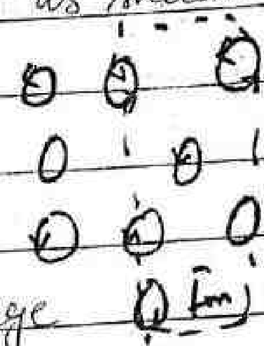
of atomic current circuits per unit volume. times the average or Effective magnetic Moment of Each circuit, one of these derivatives curl $\vec{M}$ turns out to be the Equilant transport current density. which produce the same Magnetic field as $\vec{M}$. It self it is called this magnetization current density. $\vec{J_m}$.

Magnetization Gives the over all view of matter. In the magnetized state the summation $\sum_i \vec{m_i}$ will sum to zero as a result of random orientation of the $\vec{m_i}$.

Non-Uniform Magnetization:-

If the Magnetization is non-uniform then the cancellation will not be complete.

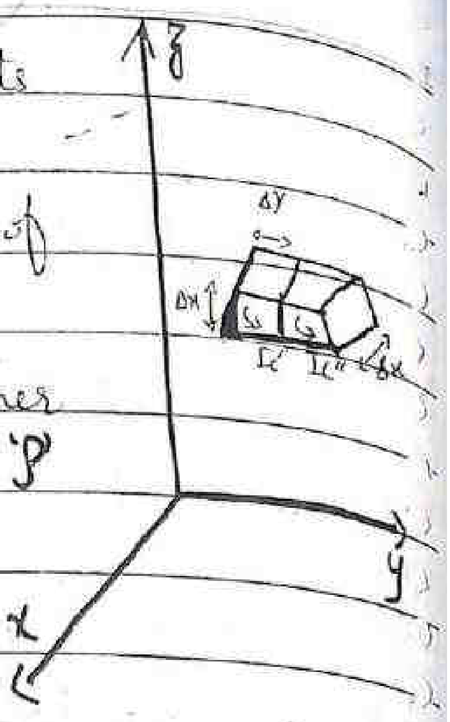
As an example of non-uniform magnetization consider the abrupt change in magnetization shown in figure as shown.



If we focus our attention on the region between the dotted lines. It is evident that there is more charge (J_m) moving down then there is moving up which we called magnetization current. Thus even though there is no charge transport there is an effective motion of charge downward. This current can produce the magnetic field.

Relation b/w $\vec{J_m}$ and $\vec{m}$:-

let us consider



we small volume Elements in a piece of magnetic Material Each Element of volume is $\Delta x, \Delta y, \Delta z$ and located next to Each other in The derivation of the 'J' is Shown in figure.

Magnetic Moment of 1^{st} Element is

$$I_c' = M_x \Delta x$$

Multiply this by Δy and Δz on both sides:

$$I_c' \Delta y \Delta z = M_x \Delta x \Delta y \Delta z \longrightarrow (a)$$

(Along y-axis)

Now Magnetic Moment of 2^{nd} Element is

$$I_c'' = \left[M_x + \frac{\partial M_x}{\partial y} \Delta y\right] \Delta x$$

Multiply by Δy and Δz.

$$\Delta y \Delta z \; I_c'' = \left[M_x + \frac{\partial M_x}{\partial y} \Delta y\right] \Delta x \; \Delta y \Delta z \longrightarrow (b)$$

(Along y-axis)

The net upward current in the middle region of the two volume

c

Element is taken by subtracting (b) from (a)

$$(\hat{I}_c' - \hat{I}_c'')\,\Delta y \Delta z = M_x \cancel{\Delta x}\,\cancel{\Delta y}\,\Delta z - M_x \Delta y \Delta x\,\Delta z - \frac{\partial M_x}{\partial y}\Delta y\,\Delta x \Delta y \Delta z$$

$$(\hat{I}_c' - \hat{I}_c'')\,\Delta y \cancel{\Delta z} = -\frac{\partial M_x}{\partial y}\Delta y \Delta x\,(\Delta y \cancel{\Delta z})$$

$$\hat{I}_c' - \hat{I}_c'' = -\frac{\partial M_x}{\partial y}\Delta x\,\Delta y$$

we next consider the Two adjescent Elements volume along x- axis and focus our attention on the y- component of the magnetization in Each cell. In The middle region of The Two cells, The net upward current is due to the circulating currents which define Magnetic Moment. Magnetization is along y- axis while change is along x- axis.

$$(I_c)_{up} = -\frac{\partial M_x}{\partial y}\Delta x\,\Delta y$$

D

These are only the circulating currents of the particular cell which give rise to the net current in the x- direction. This net current which comes about from non-uniform Magnetization is called the 'Magnetization current'. This current is not the transport current but derives as we have seen from circulatory currents i.e from atomic currents in the material. The Effective area of Each currents in Eq (a) and (b) is $\Delta x, \Delta y$.

$$(\bar{J}_m)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$$

$$(\bar{J}_m)_y = \frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x}$$

$$(\bar{J}_m)_z = \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}.$$

$$\vec{J_m} = \text{curl } \vec{M}.$$

where

$$\text{curl } \vec{M} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ M_x & M_y & M_z \end{vmatrix}$$

$$\text{curl}\,\vec{M} = \hat{i}\left(\frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}\right)$$
$$+\ \hat{j}\left(\frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial x}\right)$$
$$+\ \hat{z}\left(\frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}\right)$$

$$\text{curl}\,\vec{M} = (J_m)_x\hat{i} + (J_m)_y\hat{j} + (J_m)_z\hat{k}$$

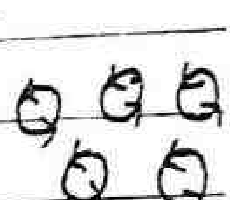
The Magnetization current density is the curl of the Magnetization.

* * * * * * * * *

Remaining part of (M.M) topic(1)

Uniform Magnetization:-

A simple model of magnetized matter as though it consisted of atomic loof currents circulating in the same direction side by side as shown in fig.



If magnetization is uniform the currents in the various loops tends to cancel Each other out and there is no net Effect current in the interaction of the material.

Maxwell Equations:-

: The Generalization of Ampere's law* is Differen The Magnetic field due to a current distribution satisfied Ampere's law

$$\text{curl}\,\vec{B} = \mu_0 \vec{J}$$ (Differential form)

Also

$$\oint \vec{B} \cdot dl = \mu_0 \oint_S \vec{J} \cdot \hat{n}\, da$$

(Integral form)

If 'H' is the Magnetic field produced then Magnetic Induction is related to Magnetic field

$$\vec{B} = \mu_0 \vec{H}$$

In Integral form the above relation become

$$\oint \mu_0 \vec{H} \cdot dl = \mu_0 \oint_S \bar{J} \cdot \hat{n}\, da$$

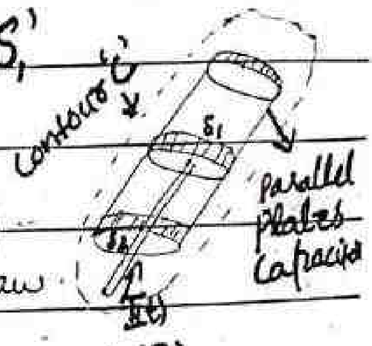
$$\cancel{\mu_0} \oint \vec{H} \cdot dl = \cancel{\mu_0} \oint_S \bar{J} \cdot \hat{n}\, da$$

$$\oint \vec{H} \cdot dl = \oint_S \bar{J} \cdot \hat{n}\, da \longrightarrow (1)$$

Now we will find the generalization of this law which is valid.

Take a cylinder having a surface S_1 and S_2 acting as a parallel capacitor plates. capacitor being charged by the constant current 'I'. If Ampere's law applied on the contour 'C'. we will Enclose in a contour 'C', for Surface 'S_1'

$$\oint_{S_1} \vec{J}\cdot\hat{n}\, da = I$$



BY Applying Ampere's law.

$$\oint_{S_1} \vec{H}\cdot dl = \oint_{S_1} \vec{J}\cdot\hat{n}_1 da = I \longrightarrow (2)$$

If on the other hand we applied Ampere's law on contour 'C' then for surface 'S_2' we have.

$$\oint_{S_2} \vec{H}\cdot dl = \oint_{S_2} J\cdot\hat{n}_2 da = 0 \longrightarrow (3)$$

on surface 'S_2', $\vec{J}$ is zero at all points current always flow on the surface (Exterior). If both Eq (2) and (3) combined Each other and thus can't both be correct If 'C' is imagined to be a great distance from the capacitor it is clear that the situation is not different from the standard Ampere's law. Eq (2) is correct because it does not depends upon capacitor. while Eq (3) depends upon distance of capacitor for their deduction. It would appear then that Eq (3) requires modification.

The proper modification can be made by noting that Eq (2) and Eq (3) give different results because the integrals on R.H.S are different

$$\oint_{S_2} \vec{J}\cdot\hat{n}_2 da - \oint_{S_1} \vec{J}\cdot\hat{n}_1 da \neq 0 \quad \rightarrow (4)$$

'S_1' and 'S_2' together form a closed surface (They Join at C) however $\hat{n}_2$ is outward drawn and $\hat{n}_1$ inward drawn.

Hence

$$\hat{n}_1 = -\hat{n}_2 = \hat{n}$$

If we take

$$n_2 = n \ \ \& \ \ n_1 = -n$$

Then Eq (4) becomes

$$\oint_{S_2} \vec{J}\cdot\hat{n} da + \oint_{S_1} \vec{J}\cdot\hat{n} da \neq 0$$

$$\oint_{S_1+S_2} \vec{J}\cdot\hat{n} da \neq 0 \quad \rightarrow (5)$$

which is just the form of the integral in the divergence Theorem. It is clear

Gaussel's law:-

$\text{div} \vec{B} = 0$ (Electric field)

Ampere's law:-

$\text{div} \vec{D} = \rho$ (Magnetic field)

where D shows displacement vector

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that the integral would vanish and thus remains the disparity between eq (2) and (3) if $\vec{J}$ were replaced by a vector $\vec{J}$, with zero divergence.

By using Divergence Theorem we get

$$\oint_{S_1+S_2} \vec{J}\cdot\hat{n}\,da = \int_V \operatorname{div}\vec{J}\cdot dv \neq 0 \rightarrow (6)$$

since the vanishing of the divergence of $\vec{J}$ ensures that surface integral vanishes we dealing with capacitor then current density is differ from conductor.

$$\vec{J}' = \underbrace{\vec{J}}_{\text{current density}} + \alpha \longrightarrow (7)$$

Where 'α' is a vector Further more 'α' must be such as to make the divergence of $\vec{J}'$ vanishes, If we take divergence of Eq (7)

$$\operatorname{div}\vec{J}' = \operatorname{div}\vec{J} + \operatorname{div}\alpha \longrightarrow (8)$$

we assume $\operatorname{div}\vec{J}' = 0$ to find its solution

$$\operatorname{div}\vec{J} + \operatorname{div}\alpha = 0$$

$$\operatorname{div}\vec{J} = -\operatorname{div}\alpha \longrightarrow (9)$$

The $\operatorname{div}\vec{J}$ may be replaced by $-\frac{\partial\rho}{\partial t}$

since differential conservation of Charge requires that

$$div \vec{J} = -\frac{ds}{dt}$$

$$div \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$div \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

As $\rho = div \vec{D}$

So, $$div \vec{J} + \frac{\partial\, div \vec{D}}{\partial t} = 0$$

$$div \vec{J} = -\frac{\partial\, div \vec{D}}{\partial t} \longrightarrow (10)$$

By comparing Eq (9) & (10)

$$div\, \alpha = \frac{\partial\, div \vec{D}}{\partial t}$$

By putting in Eq (8) The Eq becomes

$$div \vec{J}' = div \vec{J} + div \frac{\partial \vec{D}}{\partial t}$$

$$div \vec{J}' = div \left(\vec{J} + \partial \vec{D} / \partial t \right)$$

$$\vec{J}' = \vec{J} + \frac{\partial \vec{D}}{\partial t} \longrightarrow (11)$$

The dielectric displacement $\vec{D}$ is

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related to the charge density.

$$\rho = div\vec{D}$$

If $\alpha = \frac{\partial \vec{D}}{\partial t}$

$$div\vec{J}' = 0$$

$$\vec{J}' = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

As $curl\ \vec{B} = \mu_0 \vec{J}$

so $curl\ H = \vec{J}'$

Multiply by μ_0 on both sides.

$\cancel{\mu_0}\ curl\ H = \cancel{\mu_0}\vec{J}' \Rightarrow \vec{J}' = \frac{\mu_0\ curl\ H}{\mu_0}$

$curl\ \vec{H} = \vec{J}'$

Replace $\vec{J}'$ by $curl\ \vec{H}$ in Eq (11)

Then Eq becomes

$$\boxed{curl\ \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \rightarrow (12)$$

The introduction of second term in Eq (12) on right which is known as displacement current represents the large contribution of Maxwell Electromagnetic Theory. The Eq (12) shows the Ampere's law of Electromagnetic Induction.

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Maxwell's Equations And their Emperical Bases:-

The Maxwell's Equations are.

$$\text{curl } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow (a)$$

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (b)$$

$$\text{Div } \vec{D} = \rho \longrightarrow (c)$$

$$\text{div } \vec{B} = 0 \longrightarrow (d)$$ (Biot - savarts law)

Eq (a) represents the Ampere's law of Electromagnetic Induction.

Eq (b) represents the differential form of faraday's law of Electro magnetic Induction.

Eq (c) shows Gausse's law $\phi = \frac{1}{\varepsilon_0} Q$ derived from columb's law.

Eq (d) usually represents the fact that single magnetic pole have never been observed.

Electromagnetic Energy pointing vector:-

The equation which represents the Electro magnetic Energy is

$$W_E = \frac{1}{2} \int_v \vec{E} \cdot \vec{D} \, dv \rightarrow (1)$$

It can be identified with the Electro static potential Energy of the system of charges producing the Electric field. This was done by computing the work done in Establishing the field. Mean that It holds when there are Electro static charges. (charges at rest). In similar way

$$W_m = W_H = \frac{1}{2}\int_V \vec{H}\cdot\vec{B}\, dv \rightarrow (2)$$

The Energy stored in the Magnetic field. If we take Ampere's law.

$$\text{curl}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow (a)$$

Also faraday's law of Electro magnetic Induction is

$$\text{curl}\,\vec{E} = -\frac{\partial B}{\partial t} \rightarrow (b)$$

we take the scalar product of Eq (a) with $\vec{E}$ and Eq (b) with $\vec{H}$

$$\vec{E}\cdot\text{curl}\,\vec{H} = \vec{E}\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \rightarrow (c)$$

$$\vec{H}\cdot\text{curl}\,\vec{E} = -\vec{H}\cdot\frac{\partial B}{\partial t} \rightarrow (d)$$

Now subtracting (ξ) from (d)

$\vec{H}\cdot \text{curl}\vec{E} - \vec{E}\cdot\text{curl}\vec{H} = -H\cdot\frac{\partial\vec{B}}{\partial t} - E\frac{\partial\vec{D}}{\partial t} - \vec{E}\vec{J}$ $\rightarrow (3)$

To simplify the Eq (3) L.H.S we use the vector notation. According to which div of two vectors is

$$\text{div}(\vec{A}\times\vec{B}) = \vec{B}\cdot\text{curl}\vec{A} - A\cdot\text{curl}\vec{B}$$

So $\text{div}(\vec{E}\times\vec{H}) = \vec{H}\cdot\text{curl}\vec{E} - E\cdot\text{curl}\vec{H}$

Put this in Eq (3) we get

$\text{div}(\vec{E}\times\vec{H}) = -H\cdot\frac{\partial\vec{B}}{\partial t} - \vec{E}\frac{\partial\vec{D}}{\partial t} - \vec{E}\vec{J}$ $\rightarrow (4)$

If the medium to which Eq (4) is applied is linear i-e If $\vec{D}$ is proportional to $\vec{E}$ and $\vec{B}$ is propotional to $\vec{H}$, Then the derivation on R.H.S is as

As we know that

$$\vec{B} = u_0\vec{H} \rightarrow (i)$$

$$\vec{D} = \varepsilon_0\vec{E} \rightarrow (ii)$$

By Taking partial derivative w.r.t of both (i) & (ii)

$$\frac{\partial\vec{B}}{\partial t} = \frac{\partial}{\partial t}u_0\vec{H} \rightarrow (iii)$$

$$\frac{\partial\vec{D}}{\partial t} = \frac{\partial}{\partial t}\varepsilon_0\vec{E} \rightarrow (v)$$

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Take scalar product of Eq (iii) with $\vec{H}$ and (iv) with $\vec{E}$ so Eq becomes.

$$\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = \vec{H} \cdot \frac{\partial}{\partial t} \mu_0 \vec{H}$$

$$\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} = \vec{E} \cdot \frac{\partial}{\partial t} \varepsilon_0 \vec{E}$$

$$\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} (\vec{H} \cdot \mu_0) \vec{H}$$

$$\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} (\vec{E} \cdot \varepsilon_0) \vec{E}$$

$$\boxed{\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} (\vec{H} \cdot \vec{B})} \quad \because \mu_0 \cdot \vec{H} = \vec{B}$$

$$\boxed{\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} (\vec{D} \cdot \vec{E})} \quad \because \varepsilon_0 \vec{E} = \vec{D}$$

By substituting above terms in Eq (4) we get.

$$\text{div} (\vec{E} \times \vec{H}) = -\frac{1}{2} \frac{\partial}{\partial t} (\vec{H} \cdot \vec{B}) - \frac{1}{2} \frac{\partial}{\partial t} (\vec{D} \cdot \vec{E}) - \vec{J} \vec{E}$$

$$\text{div} (\vec{E} \times \vec{H}) = -\frac{1}{2} \frac{\partial}{\partial t} [\vec{H} \cdot \vec{B} - \vec{E} \cdot \vec{D}] - \vec{E} \vec{J} \quad \longrightarrow (5)$$

★ The first term on R.H.S is the time derivative of sum of Electric & magnetic energy densities.

The 2^{nd} term is in many cases Just the negative of the Joule heating rate per unit volume.
Integrating over a fixed volume 'V' bounded by the surface 'S' gives following Eq. where as volume has no Effect on time

$$\int_V div(\vec{E}\times\vec{H}) = -\frac{d}{dt}\int_V \frac{1}{2}(\vec{H}.\vec{B}+\vec{E}.\vec{D})\,dv - \int_V \vec{J}.\vec{E}\,dv \quad \rightarrow (6)$$

Apply the divergence theorem Eq then rearrange L.H.S and then whole Equation.

$$\int_V div(\vec{E}\times\vec{H}) = \int_S \vec{E}\times\vec{H}\,\hat{n}\,dv$$

put in Eq(6)

$$\int_S \vec{E}\times\vec{H}\,\hat{n}\,dv = \int_V -\frac{\partial}{\partial t}\left[\frac{1}{2}(\vec{H}.\vec{B}+\vec{E}.\vec{D})\right]dv - \int_V \vec{J}.\vec{E}\,dv$$

$$-\int_V \vec{J}.\vec{E}\,dv = \int_V \frac{\partial}{\partial t}\left[\frac{1}{2}(\vec{H}.\vec{B}+\vec{E}.\vec{D})\right]dv + \int_S (\vec{E}\times\vec{H})\,\hat{n}\,dv.$$

$$\rightarrow (7)$$

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It is clear that the $\vec{J}.\vec{E}$ term is comprised of two parts the rate of change of Electro magnetic Energy stored in 'V' and a surface integral. The L.H.S of the Eq (7) is the power transferred into the Electro magnetic field through the motion of free charge in volume 'V'.
Joule heating depends upon rate of change of Electro magnetic energy Eq Charge on the surface of Electro magnetic Energy.
* If we say that in volume 'V', there is no source of Emf (i.e battery etc) then the L.H.S of Eq (7) is -ve Eq of Equal to the minus of the Joule heat production per unit time.
In certain circumstances however the L.H.S of Eq (7) may +ve then Suppose that a charge particle is moving with certain (constant) velocity under the combined influence of Electrical Magnetic and mechanical forces

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Then the rate at which mechanical work is done by the Charge particle is thus given as.

$$\vec{P} = \vec{F}\cdot\vec{V}$$

we have Combined Effect So,

$$\vec{P} = [-q(\vec{E} + \vec{v}\times\vec{B})]\cdot\vec{v} \rightarrow (e)$$

$\because$ Power $= \frac{\text{work}}{\text{time}} = \frac{F\cdot d}{t} = \vec{F}\cdot\vec{v}$

The Eq (e) becomes

$$\vec{P} = -q\vec{E}\cdot\vec{V}$$

$\because \vec{v}\times\vec{B}\cdot\vec{V} = 0$

we Assume Magnetic field is zero

~~The current density is~~

$$P = -\Sigma_i q_i \vec{E}\cdot\vec{v}\, N_i \rightarrow (f)$$

where 'N' is no of electrons which are moving with certain velocity.

Now take current density (per unit area) as follows

$$\vec{J} = \Sigma_i N_i q_i v_i$$

$$\vec{J}\cdot\vec{E} = \Sigma_i N_i q_i \vec{v}\cdot\vec{E} \rightarrow (g) \div) \; F_m v_i = -q_i E v_i$$

$$\vec{J}\cdot\vec{E} = -\Sigma_i N_i F_m v_i$$

Now compare Eq (f) and (g)

$$\vec{P} = \vec{F}\cdot\vec{V} = \vec{E}\cdot\vec{J} = \Sigma_i N_i q \vec{v_i}\cdot E = \Sigma_i F_m\cdot v_i$$

The rate at which the mechanical

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work is done per unit volume.

$J \cdot \vec{E} = \varepsilon_i N_i q_i \vec{E} \cdot \vec{v}$

$J \cdot \vec{E} = \varepsilon_i N_i F_M \cdot \vec{v}$ $\quad \because q\vec{E} = F_M$

This power density is transferred into Electric and Magnetic fields.

The Integral of $(\vec{E} \times \vec{H})$ over a closed surface represents the rate at which Electro magnetic Energy crosses the Closed surface. The surface Integral of $(\vec{E} \times \vec{H})$ is actually known as "**Pointing vector**". Represented by '**S**'.

Pointing vector is defined as rate of flow of the local Energy per unit area.

i-e

$$\vec{S} = \vec{E} \times \vec{H}$$

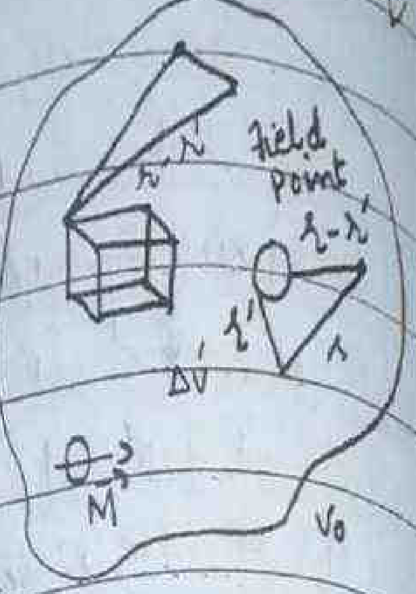
The Magnetic Field Produced by Magnetized Material:-

we take a small volume Element in magnetized material. The magnetization is written as

$$\vec{M} = \frac{\text{Sum Of Magnetic Moment (Domain M.M)}}{\text{Total volume of that Material}}$$

According to this we can write as. The Eq

$$\vec{M} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_i m_i$$



Each volume Element $\Delta V'$ of magnetized matter is characterized by the Magnetic moment. These pose now we have the magnetic moment of Small Element is

$$\Delta m = \vec{M}(x', y', z')\Delta V'$$

By using the results of "Magnetic Field at distance point," we may say that the contribution to the M.F at point (x, y, z) from Each Δm.

The Magnetic field $\vec{B}$ is then obtained as an integral over the entire volume of material V_0. Instead of calculating $\vec{B}$ directly we find $\vec{B}$ at End point Expedient to work with the vector potential $\vec{A}$ and to obtain $\vec{B}$ subsequently by means of the curl operation, The vector potential at (x', y', z') is thus given by

$$\vec{A}(x, y, z) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{M}(x', y', z') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} dv' \longrightarrow (A)$$