from equation (A) & (B) we get

$$U_1 - U_2 = T_2 - T_1$$

$$\boxed{U_1 + T_1 = U_2 + T_2}$$ Ans.

Statement:

Conservations theorems ~~for~~ of energy for a Particle States that if forces acting on Particle are Conservative. Then the total Energy of Particle remain Conserved.

Mechanics of system of Particles:

System of Particle:-

A collection of Particles form a system of Particles. Two type of forses act upon ~~each other~~. the system of particles.

(1) External Forces:-

It is the force acting upon a Particle of the system due to ~~interaction of the other Particle~~ the ~~of a the system~~ Source out side of the system.

(2) Internal Forces:-

It is the force on the Particle of the system due to the interaction of the other Particles of the system.

Considring the Particles of the

of the system, The external force acting on it is represented By $\vec{F_i^e}$ and Internal force on the Particle is

$$\sum_{j=1}^{N} F_{ji}$$

where $\vec{F_{ji}}$ is two internal forces on the ith Particle due to jth Particle.

Equation of motion of system of Particles:-

Newton 2nd law of motion Applied to ith Particle may Be Expressed as below.

$$\vec{F_i^e} + \sum_{j=1}^{n} F_{ji} = \frac{d\vec{P_i}}{dt}$$

$\boxed{\text{as } F = \frac{dP}{dt}}$

$$= m_i \frac{dv_i}{dt}$$

$$= m_i \frac{d^2\bar{r}_i}{dt^2}$$

Taking Sum over all the Particle, The above equation Become.

$$\sum_i m_i \frac{d^2 r_i}{dt^2} = \sum_i \vec{F_i^e} + \sum_i \sum_{j=1}^{N} F_{ji}$$

$$\frac{d^2}{dt^2}\left(\sum_i m_i r_i\right) = \sum \vec{F_i^e} + \sum_{j,i,N} \vec{F_{ji}}$$

$$\text{as } \sum_i \sum_{j=}^{N} F_{ji} = \sum_{j,i} F_{ji}$$

$$\sum_i \vec{F_i^e} = (F_1^e + F_2^e + \text{—} + F_n^e)$$

$$= \vec{F_e} \text{ (Total external force)}$$

$\sum_i \vec{F_{ie}}$ is the simply total External Force which may Be represented By $\vec{F_e}$. It is assume That $\vec{F_{ef}}$ obeys Newton 3rd law of motion, That mean.

$$F_{ji} = -F_{ij}$$

Then

$$F_{ji} + F_{ij} = 0$$

⇒ Then 2nd term in above equation will Be equal to zero. So the equation of motion Become as.

$$\frac{d^2}{dt^2}\left(\sum_i m_i \vec{r_i}\right) = \vec{F_e}$$

To simplify the L.H.S of the above equation, we define a vector $\vec{R}$ as follows.

$$\vec{R} = \frac{\sum_i m_i r_i}{\sum_i m_i} = \frac{\sum m_i r_i}{M}$$

$$\sum m_i r_i = M\vec{R}$$

R defines a Pt. Known as the center of mass of the system. So Now the equation of motion for system of Particle Become.

$$\frac{d^2}{dt^2}(M\vec{R}) = \vec{F_e}$$

Rearrange above

$$\boxed{M\frac{d^2R}{dt^2} = \vec{F_e}}$$

This equation means That the entire system moves as if The total external force is acting on the entire mass of the system which is Considered to Be Concentrated at the Center of mass of the system. Thus The Problem of the system of forces, [illegible]

Converted to that of single force.
It also indicates that purely internal Forces have no effect on the motion Center of mass. Hence in case of Exploding shell. The fragments moves such that the Center of mass will still Be same, that of Composite shell.

Linear Momentum of system of Particle

Linear momentum of the system will Be equal to the sum of linear momenta of all the Particles.

$$\vec{P} = \vec{P_1} + \vec{P_2} + \vec{P_3} + \cdots + P_n$$

$$\vec{P} = \sum m_i v_i$$

$$= \sum m_i \frac{dr_i}{dt}$$

$$= \frac{d}{dt}\left(\sum m_i r_i\right)$$

As $\sum m_i r_i = MR$ (Center of mass)

$$= \frac{d}{dt}(MR)$$

$$\boxed{\vec{P} = M\frac{dR}{dt}}$$

Conservation Theorm for linear momentum of system

If The Total External force on the system ~~of~~ is Zero. The total linear momentum will Be Conserved.

As

$$P = M\frac{d\vec{R}}{dt}$$

$$\frac{dP}{dt} = M\frac{d^2R}{dt^2}$$

As $\frac{dP}{dt} = \vec{F_e}$

And $F_e = 0$ (External for is zero)

$$\frac{dP}{dt} = 0$$

Integrating above we get

$$\boxed{P = \text{Constant}}$$

Drive the expression for Torque acting on system of Particles from angular momentum:-

for the system of Particles Total angular momentum will Be;

$$\bar{L} = \sum \bar{r_i} \times \bar{p_i}$$

$$\bar{N} = \frac{d\bar{L}}{dt} = \frac{d}{dt}(\sum \bar{r_i} \times \bar{p_i})$$

$$\bar{N} = \sum \frac{dr_i}{dt} \times p_i + \sum r_i \times \frac{dp_i}{dt}$$

$$= \sum v_i \times p_i + \sum r_i \times \frac{dp_i}{dt}$$

$$= \sum v_i \times m v_i + \sum r_i \times \frac{dp_i}{dt}$$

$\because v \times v = 0$

$$= 0 + \sum r_i \times \frac{dp_i}{dt}$$

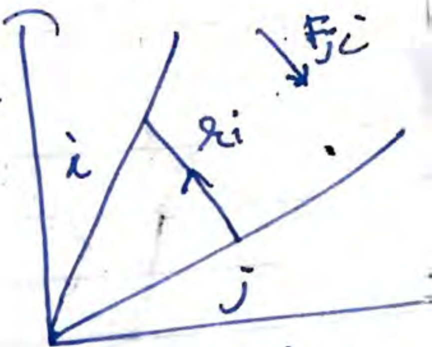
$$= \sum r_i \times (F_{ie} + \sum_j F_{ji})$$

$= \sum r_i \times F_{ji}$

Now above Expression Contain sum of Pair of terms as

$= r_i \times F_{ji} + r_j \times F_{ij}$

$= (r_i - r_j) \times F_{ji}$



$= r_{ij} \times F_{ji}$

$= 0$

if The internal forces Between two Particles in addition to Being equal and opposite also act along the line joining The Particle Then

$r_{ij} \times F_{ji} = 0$

It should Be noted that This assumption is not true in case of forces acting on moving charge Particles This is Beacuse These forces are velocity dependent and do not act along The line joining The Particles

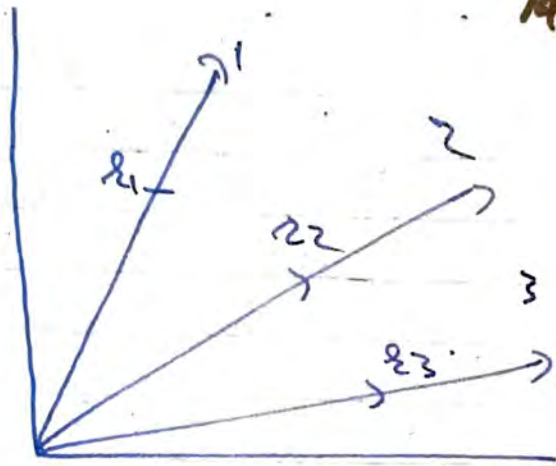
The time the total change acting on system of Particle is

$\bar{N} = \sum r_i \times F_i^e$

= Total ~~Charge~~ Torque on system of Particles due to the external force.

$$= r_1 \times (F_{11} + F_{21} + F_{31}) + r_2 \times (F_{12} + F_{22} + F_{32}) + r_3 \times (F_{13} + F_{23} + F_{33})$$

(Inter actions among Three Particles)



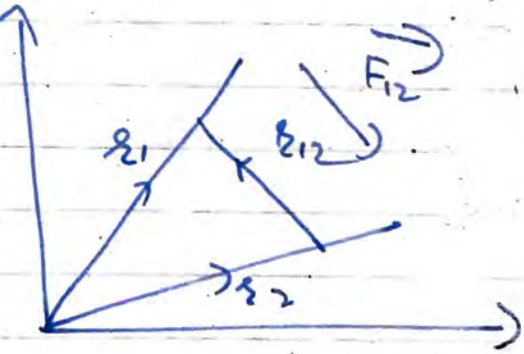
$$= \sum_{i=1}^{3} r_i \times F_{ji}$$

$$= r \times (F_{21} + F_{31}) + r_2 (F_{12} + F_{32}) + r_3 \times (F_{13} + F_{23})$$

As we know $F_{12} = -F_{21} \Rightarrow$

$$= (r_1 - r_2) \times F_{21} + (r_1 - r_3) \times F_{31} + (r_2 - r_3) \times F_{32}$$

$= r_{12} \times F_{21} = 0$ as They act at the same line.



Angular momentum of system of Particles in terms of Center of mass

Let m_i & r_i represent The mass and Position vector of ith Particle with refrence to Pt. O.