

$$= \frac{q}{c} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{i} + \frac{q}{c} \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{j} + \frac{q}{c} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \hat{k}$$

(As we put E_x, E_y, E_z in equations)

$$= \frac{q}{c} (0 + 0 + 0)$$

$$\text{Curl } \vec{E} = \frac{q}{c} (0) = 0 \quad \square$$

\Rightarrow This curl is zero. Hence the electric field of force is conservative field of force.

In similar way we can prove that Gravitational is conserved also

*** Gravitational field is conserved:-**

we know that Gravitational Force is given by:

$$F = -\frac{G m_1 m_2}{r^2} \hat{r} \quad (\text{-ve sign due to attractive force})$$

$$\vec{F} = -\frac{G m_1 m_2}{r^3} \vec{r}$$

$$F = -\frac{G m_1 m_2}{r^3} \vec{r}$$

As we know that

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$$

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

$$|\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$$

$$r = (x^2 + y^2 + z^2)^{3/2}$$

Then we can write as follows

$$(F_x i + F_y j + F_z k) = -G m_1 m_2 \left(\frac{x_i + y_j + z_k}{(x^2 + y^2 + z^2)^{3/2}} \right)$$

$$(F_x i + F_y j + F_z k) = -G m_1 m_2 \left(\frac{x_i}{(x^2 + y^2 + z^2)^{3/2}} + \frac{y_j}{(x^2 + y^2 + z^2)^{3/2}} + \frac{z_k}{(x^2 + y^2 + z^2)^{3/2}} \right)$$

Comparing the coefficient of i, j, k we get

$$F_x = \frac{-G m_1 m_2 x}{(x^2 + y^2 + z^2)^{3/2}}$$

$$F_y = \frac{-G m_1 m_2 y}{(x^2 + y^2 + z^2)^{3/2}}$$

$$F_z = \frac{-G m_1 m_2 z}{(x^2 + y^2 + z^2)^{3/2}}$$

Now we take the curl $F \Rightarrow$

$$\text{Curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\text{Curl } \vec{F} = i \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + j \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + k \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

$$= i(0) + j(0) + k(0)$$

$$\text{Curl } F = 0$$

Hence it shows that Gravitational field is conservative field.

$\nabla \cdot \text{Grad } U$: Grade U is the maximum increase of scalar at a given pt. along a particular field. $\text{grad } U$ is a vector quantity.

$\nabla \cdot$ Show that $\vec{F} = -\text{grad } U$.
 If U is a scalar point function. Then

$$\text{grad } U = \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k}$$

Now taking curl of $\text{grad } U \Rightarrow$

$$\text{Curl}(\text{grad } U) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} & \frac{\partial U}{\partial z} \end{vmatrix}$$

$$\begin{aligned}
 &= \sum_i \left(\frac{\partial}{\partial y} \left(\frac{\partial U}{\partial z} \right) - \frac{\partial}{\partial z} \left(\frac{\partial U}{\partial y} \right) \right) \\
 &= \sum_i \left(\frac{\partial^2 U}{\partial y \partial z} - \frac{\partial^2 U}{\partial z \partial y} \right)
 \end{aligned}$$

As it's Perfect Potential

$$\Rightarrow \frac{\partial^2 U}{\partial y \partial z} = \frac{\partial^2 U}{\partial z \partial y} \Rightarrow$$

$$= \sum_i \left(\frac{\partial^2 U}{\partial y \partial z} - \frac{\partial^2 U}{\partial y \partial z} \right)$$

$$\text{Curl}(\text{grad } U) = 0 \quad \text{--- (1)}$$

In previous article we have shown that $\text{Curl } \vec{F} = 0$ --- (2)

From eq (1) & (2) we get

$$\text{Curl } F = \text{Curl}(\text{grad } u)$$

or

$$F = \text{grad } u$$

In order that scalar function has a physical meaning it assign a -ve sign with it. i.e.

$$\boxed{F = -\text{grad } u}$$

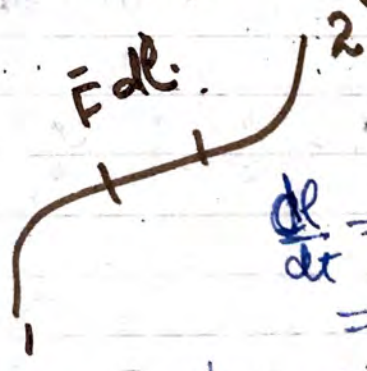


$$E = -\text{grad } v$$

v is Potential in field.

Conservative Theorem of Energy:

$$W_{12} = \int_1^2 F \cdot d\vec{r}$$



$$v = \frac{ds}{dt}$$

Vel. $\frac{dr}{dt}$

$$\frac{dr}{dt} = v$$
$$\frac{dr}{dt} = v \frac{dt}{dt}$$

$$W_{12} = \int_1^2 m \frac{dv}{dt} \cdot v dt$$

$$= \frac{m}{2} \int_1^2 \left(\frac{dv}{dt} \cdot v \right) dt$$

$$W_{12} = \frac{m}{2} \int_1^2 \frac{d(v \cdot v)}{dt} dt$$

$$\frac{d(v \cdot v)}{dt}$$
$$= \frac{dv}{dt} \cdot v + v \cdot \frac{dv}{dt}$$

$$= 2v \cdot \frac{dv}{dt}$$

$$\frac{d(v \cdot v)}{dt} = 2v \cdot \frac{dv}{dt}$$

$$W_{12} = \frac{m}{2} \int_1^2 \frac{d}{dt} (v^2) dt$$

$$= \frac{m}{2} \left| v^2 \right|_1^2$$

W_{12}

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

Now

Put

$$T_1 = \frac{1}{2} m v_1^2$$

$$T_2 = \frac{1}{2} m v_2^2$$

$$W_{12} = T_2 - T_1$$

(A)

————— A ———

$$W_{12} = \int_1^2 -\text{grad } u \cdot d\mathbf{l}$$

$$= - \int_1^2 \left(\frac{du}{dx} \mathbf{i} + \frac{du}{dy} \mathbf{j} + \frac{du}{dz} \mathbf{k} \right) \cdot (dx \mathbf{i} + dy \mathbf{j} + dz \mathbf{k})$$

$$\text{As: } \text{grad } u = \frac{du}{dx} \mathbf{i} + \frac{du}{dy} \mathbf{j} + \frac{du}{dz} \mathbf{k}$$

$$d\mathbf{l} = dx \mathbf{i} + dy \mathbf{j} + dz \mathbf{k}$$

$$W_{12} = - \int_1^2 \left(\frac{du}{dx} dx + \frac{du}{dy} dy + \frac{du}{dz} dz \right)$$

$$W_{12} = - \int_1^2 (du)$$

$$= - \left(u \right|_1^2$$

$$W_{12} = - (u_2 - u_1)$$

$$= -u_2 + u_1 = u_1 - u_2$$

$$W_{12} = u_1 - u_2 \quad \text{(B)}$$

From equation (A) & (B) we get "

$$U_1 - U_2 = T_2 - T_1$$
$$U_1 + T_1 = U_2 + T_2$$

Ans:

Statement:

Conservation Theorem of energy for a Particle states that if forces acting on Particle are conservative. Then the total Energy of Particle remain conserved.

* Mechanics of system of Particles:-

System of Particle:-

A collection of Particles form a system of Particles. Two type of forces act upon each other. the system of particles.

(1) External Forces:-

It is the force acting upon a Particle of the system due to the ~~interaction of the other Particle of the system~~ source out side of the system.

(2) Internal Forces:-

It is the force on the Particle of the system due to the interaction of the other Particles of the system.
Considering the Particles of the