perpendicular ~~bar~~

Moment Arm:- It is the distance Between The line of action of force and the axis of rotation. In fig. moment arm is e or $r\sin\theta$.

Angular Momentum:- ($\overline{L}$):- Moment of the linear Momentum is Called angular momentum

$\overline{L} = \vec{r} \times \overline{P}$

where $\overline{P}$ is a linear momentum.

$= m(\vec{r} \times \overline{v})$

Relation Between ($\overline{N}$ & $\overline{L}$):-

We Know That $\overline{L} = \overline{r} \times \overline{P}$

and

$$\frac{d\overline{L}}{dt} = \frac{d}{dt}(\overline{r} \times \overline{P}) \qquad P = mv$$

$$= \frac{d}{dt}(\vec{r} \times m\overline{v})$$

$$= m\frac{d}{dt}(\vec{r} \times \overline{v})$$

$$= m\left(\frac{dr}{dt} \times v + r \times \frac{dv}{dt}\right)$$

$$= m(v \times v + r \times a)$$

as $v \times v = 0$

$$\Rightarrow = m(0 + r \times a)$$

$$= m(r \times a)$$

$$\frac{d\overline{L}}{dt} = r \times ma \qquad F = ma$$

$$= r \times F$$

$$\boxed{\frac{d\bar{L}}{dt} = \bar{r} \times \bar{F}}$$ which is Proof

" This Torque is equal to rate of change of angular momentum "

$$\bar{N} = \frac{dL}{dt}.$$

EXP:- A Particle moves in a force field given by

$$\vec{F} = r^2 \vec{r}$$, r is Position vector of $\vec{r}$. Prove that angular momentum is conserved in this case, we know

$$\frac{d\bar{L}}{dt} = \bar{N} \Rightarrow L = \text{constant}$$

As

$$\frac{d\bar{L}}{dt} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times r^2 \vec{r}$$

$$\frac{d\bar{L}}{dt} = 0$$

So L is constant or conserved.

Conservation Theorem for A Single Particle:-

(1) Linear Momentum:- If the total force $\vec{F}$ acting on a particle is zero, then the linear momentum P is conserved i.e

$$\vec{F} = 0$$

$$\frac{dP}{dt} = F = 0$$

$$\Rightarrow P = \text{Constant}$$

(ii) Angular Momentum:- If The Total Torque $\bar{N}$ acting on a particle is zero. Then angular momentum L is conserved

$$\bar{N} = 0$$

$$\Rightarrow \frac{dL}{dt} = 0$$

Integrating Both sides we get

$$\Rightarrow L = \text{Constant}$$

Some defination:-

(i) Point function:- A function which depend upon the location of a Pt eg, electric Intensity, Potential etc.

(ii) Scaler Pt. function:- A Pt. function which has magnitude only eg Potental and Potental energy etc.

(iii) vector Pt. function:- A Pt-function which has magnitude and direction also is called vector Pt function.

(iv) Curl of vector:-

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\bar{\nabla} \times \bar{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{i} + \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\hat{j} + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

* Stock's Theorm:-

stock's Theorm is mathmatically written as:

$$\oint \bar{F} \cdot d\bar{l} = \int_S \text{Curl } F \cdot \hat{n} \, d\vec{s}$$

* Conservative field of force:-

we can state it in two way

(I) if the force field is such that the work done on a particle W_{12} is same for any physicaly passiable path, between Pts I & II. Then the force field is said to be conservative.

(II) An Alternate description of the conservative system is obtain by imaginary a particle being taken from Pt (1) to (2) by one passiable path and then it is re-turned back to Pt (1) by another path. If the work done around such closed path is zero. Then the force called conservative. Mathmatically for field to Be Conservative

$$\oint \bar{F} \cdot d\bar{l} = 0$$

Physically it means That in a non-dissipative field, The energy spent in moving a Particle from (1) to (2) is equal to the Energy gain in Bringing the Particle Back from (2) to (1). Thus the net change in Energy in a closed loop is zero. if a dissipative force as friction is Present, The $\bar{F} \cdot d\bar{l}$ is always +ve i.e. extra work will have to Be done to over come friction. Then Thus the loop integral $\oint \bar{F} \cdot d\bar{l}$ Can't vanishes. Applying stocks's Theorm

$$\oint \bar{F} \cdot d\bar{l} = \int_S \text{Curl}\, \bar{F} \cdot n\, ds = 0$$

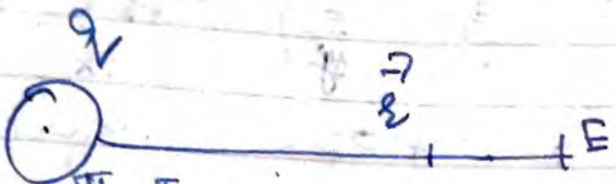
where $n\, ds$ is the element of the Area of Surface Covered By the loop. This equation will Be true only if Curl $\vec{F} = 0$. Thus Curl of a vector determine where the field is Conservative or not.

Critaria for field to Be Conserved :-

(i) Field Should Be non-dissipative field.

(ii) Curl F Should Be equal to Zero.

* Show That electrical field is Conservative:-



we Know That

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

we also know That

$$\vec{E} = E_x i + E_y j + E_z k$$

as

$$\bar{E} = \frac{q}{c} \frac{\hat{r} r}{r^3}$$

$$\bar{E} = \frac{q}{c} \frac{\vec{r}}{r^3}$$

as we Know That

$$r = xi + yj + zk$$

$$|r| = \sqrt{x^2 + y^2 + z^2}$$

$$r^3 = (x^2 + y^2 + z^2)^{3/2}$$

$$E = \frac{q}{c} \frac{xi + yj + zk}{(x^2 + y^2 + z^2)^{3/2}}$$

$$E = \frac{q}{c} \left(\frac{xi}{(x^2+y^2+z^2)^{3/2}} + \frac{yj}{(x^2+y^2+z^2)^{3/2}} + \frac{zk}{(x^2+y^2+z^2)^{3/2}} \right.$$

Now Taking curl of E

$$\text{curl } E = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \frac{q}{c}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)i + j\,\frac{q}{c}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right)$$

$$+ k\,\frac{q}{c}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$ (As we put E_x, E_y & E_z in equations)

$$= \frac{q}{c}(0 + 0 + 0)$$

$$\text{Curl } \vec{E} = \frac{q}{c}(0) = 0$$

⇒ This curl is zero. Hence the electric field of force is conserved field of force.

In similar way we can prove that Gravational is conserved also

Gravational field is conserved:-

We know that Gravational Force is given By;

$$F = -G\frac{m_1 m_2}{r^2}\hat{r}$$ (-ve singe due to attractive force)

$$\vec{F} = -\frac{G m_1 m_2}{r^3}\hat{r}r$$

$$F = -\frac{G m_1 m_2}{r^3}\vec{r}$$

As we know that

$$F = F_x i + F_y j + F_z k$$

$$\vec{r} = xi + yj + zk$$

$$|r| = (x^2 + y^2 + z^2)^{1/2}$$

$$r^3 = (x^2 + y^2 + z^2)^{3/2}$$