42-5 DOUBLE-SLIT INTERFERENCE AND DIFFRACTION COMBINED

In our analysis of double-slit interference (Section 41-2) we assumed that the slits were arbitrarily narrow—that is, that $a \ll \lambda$. For such narrow slits, the central part of the screen on which the light falls is uniformly illuminated by the diffracted waves from each slit. When such waves interfere, they produce interference fringes of uniform intensity.

In practice, for visible light, the condition $a \ll \lambda$ is usually not met. For such relatively wide slits, the intensity of the interference fringes formed on the screen is *not* uniform. Instead, the intensity of the fringes varies within an envelope due to the diffraction pattern of a single slit.

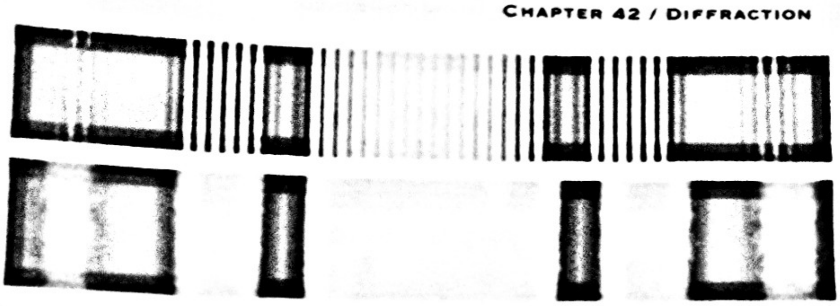

FIGURE 42-16. (*a*) Interference fringes for a double-slit system in which the slit width is not negligible in comparison with the wavelength. (*b*) The diffraction pattern of a single slit of the same width. Note that the diffraction pattern modulates the intensity of the interference fringes, as shown in part (*a*).

The effect of diffraction on a double-slit interference pattern is illustrated in Fig. 42-16, which compares the double-slit pattern with the diffraction pattern produced by a single slit of the same width as each of the double slits. You can see from Fig. 42-16*a* that the diffraction does indeed provide an intensity envelope for the more closely spaced double-slit interference fringes.

Let us now analyze the combined interference and diffraction pattern of Fig. 42-16*a*. The interference pattern for two infinitesimally narrow slits is given by Eq. 41-13 or, with a small change in notation,

$$I_{\theta,\text{int}} = I_{\text{m,int}} \cos^2 \beta, \qquad (42\text{-}13)$$

where

$$\beta = \frac{\pi d}{\lambda} \sin \theta, \qquad (42\text{-}14)$$

in which d is the distance between the centerlines of the slits.

The intensity for the diffracted wave from either slit is given by Eq. 42-8 or, again with a small change in notation,

$$I_{\theta,\text{dif}} = I_{\text{m,dif}} \left(\frac{\sin \alpha}{\alpha} \right)^2, \qquad (42\text{-}15)$$

where

$$\alpha = \frac{\pi a}{\lambda} \sin \theta. \qquad (42\text{-}16)$$

We find the combined effect by regarding $I_{\text{m,int}}$ in Eq. 42-13 as a variable amplitude, given in fact by $I_{\theta,\text{dif}}$ of Eq. 42-15. This assumption, for the combined pattern, leads to

$$I_\theta = I_\text{m} (\cos \beta)^2 \left(\frac{\sin \alpha}{\alpha} \right)^2, \qquad (42\text{-}17)$$

in which we have dropped all subscripts referring separately to interference and diffraction. Later in this section we derive this result using phasors.

Let us express this result in words. At any point on the screen the available light intensity from each slit, considered separately, is given by the diffraction pattern of that slit (Eq. 42-15). The diffraction patterns for the two slits, again considered separately, coincide because parallel rays in Fraunhofer diffraction are focused at the same spot. Because the two diffracted waves are coherent, they interfere.

The effect of interference is to redistribute the available energy over the screen, producing a set of fringes. In Section 41-2, where we assumed $a \ll \lambda$, the available energy was virtually the same at all points on the screen so that the interference fringes had virtually the same intensities (see Fig. 41-9). If we relax the assumption that $a \ll \lambda$, the available energy is *not* uniform over the screen but is given by the diffraction pattern of a slit of width a. In this case the interference fringes have intensities that are determined by the intensity of the diffraction pattern at the location of a particular fringe. Equation 42-17 is the mathematical expression of this argument. This is especially clear in Fig. 42-17, which shows (*a*) the "interference factor" in Eq. 42-

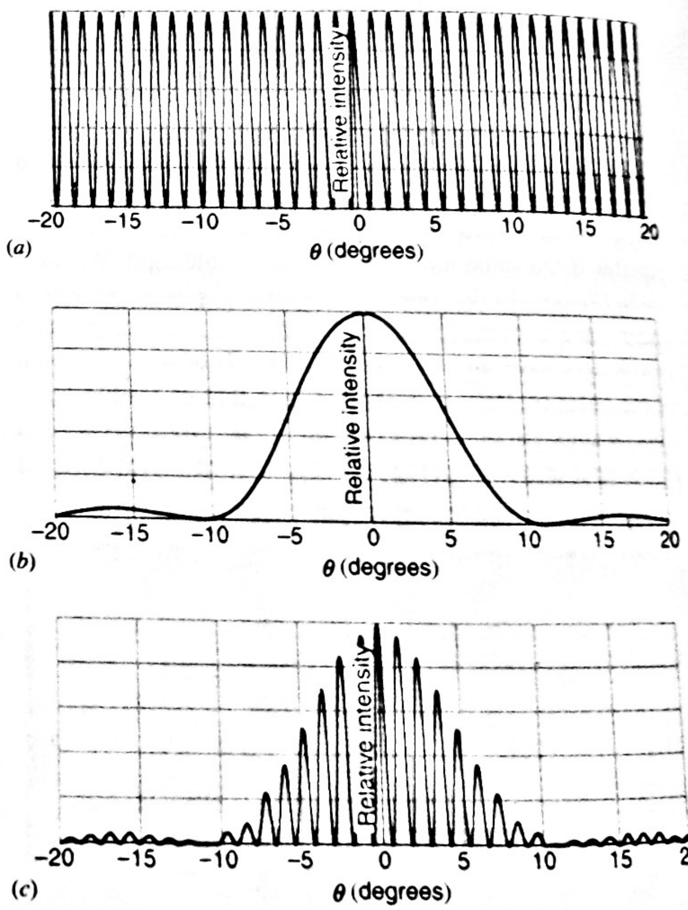

FIGURE 42-17. (*a*) Interference fringes that would be produced by a double slit of vanishingly narrow widths. (*b*) The diffraction pattern for a slit of finite width. (*c*) The pattern of interference fringes formed by two slits of the same width as that of (*b*). This pattern is equivalent to the product of the curves shown in (*a*) and (*b*). Compare Fig. 42-16*a*.