43-3 DISPERSION AND RESOLVING POWER

The ability of a grating to produce spectra that permit precise measurement of wavelengths is determined by two intrinsic properties of the grating: (1) the separation $\Delta\theta$ between spectral lines that differ in wavelength by a small amount $\Delta\lambda$ and (2) the width or sharpness of the lines.

In Sample Problem 43-2, we calculated the angular separation between the closely spaced lines of the yellow sodium doublet, for which $\Delta\lambda = 0.59$ nm. We found in that case a separation of $\Delta\theta = 0.014°$ between the first-order principal maxima of these lines. The angular separation $\Delta\theta$ per unit wavelength interval $\Delta\lambda$ is called the *dispersion D* of the grating, or

$$D = \frac{\Delta\theta}{\Delta\lambda}. \tag{43-7}$$

For lines of nearly equal wavelengths to appear as widely separated as possible, we would like our grating to have the largest possible dispersion.

To see what physical property of the grating determines its dispersion, we differentiate Eq. 43-1 ($d \sin \theta = m\lambda$), treating θ and λ as variables, which gives

$$d \cos \theta \, d\theta = m \, d\lambda,$$

or, in terms of small differences instead of differentials,

$$d \cos \theta \, \Delta\theta = m \, \Delta\lambda. \tag{43-8}$$

The dispersion D is given by $\Delta\theta/\Delta\lambda$, or

$$D = \frac{m}{d \cos \theta}. \tag{43-9}$$

The dispersion increases as the spacing between the slits decreases. We can also increase the dispersion by working at higher order (large m), as Fig. 43-10 illustrates. Note that the dispersion does not depend on the number of rulings.

Resolving Power

If a grating produces lines of large width, then the maxima of spectral lines of closely spaced wavelengths may overlap, making it difficult to determine whether such lines have one or more components and to measure the wavelengths of the lines to high precision. We therefore want to select a grating that produces the narrowest possible lines. is Better

We obtain a reasonable measure of the ability to resolve nearby lines of different wavelengths by applying Rayleigh's criterion (see Section 42-4): if the maximum of one line falls on the first minimum of its neighbor, we should be able to resolve the lines. In Section 43-1, we defined the width of a spectra line in just that way, as the angular interval $\delta\theta$ from the maximum to the first minimum. The limit of resolution of the grating occurs when two lines in the spectrum are separated by a wavelength interval $\Delta\lambda$ such that the difference $\delta\theta$ between their angular positions is given by Eq. 43-6. We define the *resolving power* R of the grating as

$$R = \frac{\lambda}{\Delta\lambda}. \tag{43-10}$$

If the lines are to be narrow ($\delta\theta$ is small), then the corresponding wavelength interval $\Delta\lambda$ must be small, and the resolving power must be large. We should therefore choose a grating with the largest R.

To find the physical property of the grating that determines R, let us solve Eq. 43-8 for the spacing $\Delta\theta$ between nearby lines and (using Rayleigh's criterion) set this result equal to the width $\delta\theta$ of the line, given by Eq. 43-6 as the spacing between the maximum and first minimum. This gives

$$\frac{m\,\Delta\lambda}{d \cos \theta} = \frac{\lambda}{Nd \cos \theta},$$

and solving for R ($=\lambda/\Delta\lambda$) gives

$$R = Nm. \tag{43-11}$$

TABLE 43-1 Properties of Three Gratings[a]

Grating	N	d (nm)	θ	R	D (10^{-4} rad/nm)
A	5,000	10,000	2.9°	5,000	1.0
B	5,000	5,000	5.7°	5,000	2.0
C	10,000	10,000	2.9°	10,000	1.0

[a] For λ = 500 nm and m = 1.

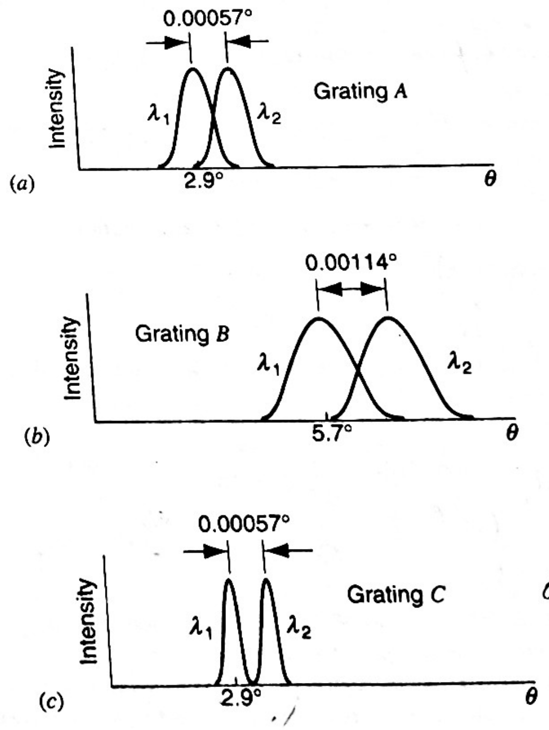
The resolving power, like the dispersion, increases with the order number. Unlike the dispersion, R depends on the number of lines N but is independent of their separation d. To maximize the resolving power, we choose a grating with the largest number of lines. For a given slit spacing d, the grating with the greatest total width has the greatest resolving power (that is, it produces the sharpest spectral lines).

Dispersion and resolving power measure different aspects of a diffraction grating's ability to produce cleanly separated lines. Consider, for example, three gratings A, B, and C whose properties are listed in Table 43-1. Suppose that the gratings are illuminated with light consisting of a doublet of lines at 500 nm separated by an interval $\Delta\lambda$ = 0.10 nm. We have chosen the properties of grating A such that the two lines of the doublet in the first-order maximum are just at the limit of resolution; that is, the maximum of one line falls on the minimum of the other, as shown in Fig. 43-11a. Grating B has twice the dispersion of A but the same resolving power, and it produces the spectrum shown in Fig. 43-11b. In effect, all angular intervals are scaled by

Grating A

B

C



doubled

FIGURE 43-11. The intensity pattern of two lines at λ = 500 nm separated by $\Delta\lambda$ = 0.10 nm, produced by the three gratings of Table 43-1. Grating B has the largest dispersion and grating C the largest resolving power.

a factor of 2, including the angular width and angular separation of the peaks. If our measurement with grating *A* had been limited by our ability to determine small angular intervals, changing to grating *B* would improve the measurement.

Grating *C* has twice the resolving power of *A* but the same dispersion. The peaks in Fig. 43-11*c* appear with the same angular separation as those in Fig. 43-11*a*, but with smaller widths. The maximum of one peak now clearly falls outside the first minimum of the other, and the two lines are more clearly distinguished from one another using grating *C*.

The total widths of the three gratings, equal to the product Nd, are 50 mm for grating *A*, 25 mm for grating *B*, and 100 mm for grating *C*. Note from Fig. 43-11 that the peak widths depend inversely on the grating width, as suggested by Eq. 43-6.