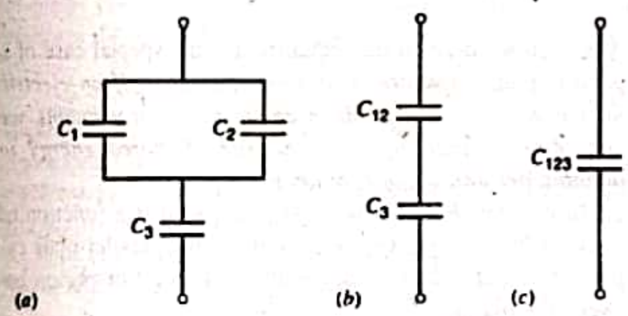

FIGURE 30-9. Sample Problem 30-5. (*a*) A combination of three capacitors. (*b*) The parallel combination of C_1 and C_2 has been replaced by its equivalent, C_{12}. (*c*) The series combination of C_{12} and C_3 has been replaced by its equivalent, C_{123}.

$C_2 = 5.3\ \mu F$, and $C_3 = 4.5\ \mu F$. (*b*) A potential difference $\Delta V =$ 12.5 V is applied to the terminals in Fig. 30-9*a*. What is the charge on C_1?

Solution (*a*) Capacitors C_1 and C_2 are in parallel. From Eq. 30-16, their equivalent capacitance is

$$C_{12} = C_1 + C_2 = 12.0\ \mu F + 5.3\ \mu F = 17.3\ \mu F.$$

In Fig. 30-9*b*, C_1 and C_2 have been replaced by their parallel combination, C_{12}. As the figure shows, C_{12} and C_3 are in series. From Eq. 30-21, the final equivalent combination (see Fig. 30-9*c*) is found from

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3\ \mu F} + \frac{1}{4.5\ \mu F} = 0.280\ \mu F^{-1},$$

or

$$C_{123} = \frac{1}{0.280\ \mu F^{-1}} = 3.57\ \mu F.$$

(*b*) We treat the equivalent capacitors C_{12} and C_{123} exactly as we would real capacitors having that capacitance. The charge on C_{123} in Fig. 30-9*c* is then

$$q_{123} = C_{123}\,\Delta V = (3.57\ \mu F)(12.5\ V) = 44.6\ \mu C.$$

This same charge exists on each capacitor in the series combination of Fig. 30-9*b*. The potential difference across C_{12} in that figure is then

$$\Delta V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\ \mu C}{17.3\ \mu F} = 2.58\ V.$$

This same potential difference appears across C_1 in Fig. 30-9*a*, so that

$$q_1 = C_1\,\Delta V_1 = (12\ \mu F)(2.58\ V) = 31\ \mu C.$$

30-5 ENERGY STORAGE IN AN ELECTRIC FIELD

An important use of capacitors is to store electrostatic energy in applications ranging from flash lamps to laser systems (see Fig. 30-10), both of which depend for their operation on the charging and discharging of capacitors.

In Section 28-2 we showed that any charge configuration has a certain *electric potential energy U*, equal to the

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30-5 ENERGY STORAGE IN AN ELECTRIC FIELD

An important use of capacitors is to store electrostatic energy in applications ranging from flash lamps to laser systems (see Fig. 30-10), both of which depend for their operation on the charging and discharging of capacitors.

In Section 28-2 we showed that any charge configuration has a certain *electric potential energy U*, equal to the work W (which may be positive or negative) that is done by an external agent that assembles the charge configuration from its individual components, originally assumed to be infinitely far apart and at rest. This potential energy is similar to that of mechanical systems, such as a compressed spring or the Earth–Moon system.

FIGURE 30-10. This bank of 10,000 capacitors at the Lawrence Livermore National Laboratory stores 60 MJ of electric energy and releases it in 1 ms to flashlamps that drive a system of lasers. The installation is part of the Nova project, which is attempting to produce sustained nuclear fusion reactions.

For a simple example, work is done when two equal and opposite charges are separated. This energy is stored as electric potential energy in the system, and it can be recovered as kinetic energy if the charges are allowed to come together again. Similarly, a charged capacitor has stored in it an electrical potential energy U equal to the work W done by the external agent as the capacitor is charged. This energy can be recovered if the capacitor is allowed to discharge. Alternatively, we can visualize the work of charging by imagining that an external agent pulls electrons from the positive plate and pushes them onto the negative plate, thereby bringing about the charge separation. Normally, the work of charging is done by a battery, at the expense of its store of chemical energy.

Suppose that at a time t a charge q' has already been transferred from one plate to the other. The potential difference

$\Delta V'$ between the plates at that moment is $\Delta V' = q'/C$. If an increment of charge dq' is now transferred, the resulting small change dU in the electric potential energy is, according to Eq. 28-9 ($\Delta V = \Delta U/q_0$),

$$dU = \Delta V' dq' = \frac{q'}{C}\, dq'.$$

If this process is continued until a total charge q has been transferred, the total potential energy is

$$U = \int dU = \int_0^q \frac{q'}{C}\, dq' \tag{30-23}$$

or

$$U = \frac{q^2}{2C}. \tag{30-24}$$

From the relation $q = C\,\Delta V$ we can also write this as

$$U = \tfrac{1}{2}C(\Delta V)^2 \tag{30-25}$$

Where does this energy reside? Equations 30-24 and 30-25 do not give us a direct answer, but we can determine the location of the stored energy by reasoning as follows. Suppose we have an isolated parallel-plate capacitor (that is, not connected to a battery) that carries a charge q. Without changing q, we pull the plates apart until their separation is twice as large as it was initially. According to Eq. 30-5, if the plate separation d becomes twice as large, the capacitance becomes only half as large. Equation 30-24 shows that if C becomes half as large, the stored energy doubles. Now in pulling the plates apart we have not changed the capacitor plates, so it would not be reasonable to conclude that the extra energy is stored there. What we have done is to double the volume of the space between the plates, and since the energy has also doubled it seems reasonable to conclude that this electric potential energy resides in the volume between the plates. More specifically, *the energy is stored in the electric field that is present in this region.*

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value for all points between the plates. Based on our conclusion that the energy resides in the field, it follows that the *energy density* u, which is the stored energy per unit volume, should also be the same everywhere between the plates. u is given by the stored energy U divided by the volume Ad, or

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}C(\Delta V)^2}{Ad} \tag{30-26}$$

Substituting the relation $C = \epsilon_0 A/d$ (Eq. 30-5) leads to

$$u = \frac{\epsilon_0}{2}\left(\frac{\Delta V}{d}\right)^2. \tag{30-27}$$

However, $\Delta V/d$ is the electric field E, so that

$$u = \tfrac{1}{2}\epsilon_0 E^2. \tag{30-28}$$

Although we derived this equation for the special case of a parallel-plate capacitor, it is true in general. *If an electric field* $\vec{E}$ *exists at any point in empty space (a vacuum), we can think of that point as the site of stored energy in amount, per unit volume, of* $\frac{1}{2}\epsilon_0 E^2$.

In general, E varies with location, so u is a function of the coordinates. For the special case of the parallel-plate capacitor, E and u do not vary with location in the region between the plates.

SAMPLE PROBLEM 30-6. A 3.55-μF capacitor C_1 is charged to a potential difference $\Delta V_0 = 6.30$ V, using a battery. The charging battery is then removed, and the capacitor is connected as in Fig. 30-11 to an uncharged 8.95-μF capacitor C_2. After the switch S is closed, charge flows from C_1 to C_2 until an equilibrium is established, with both capacitors at the same potential difference ΔV. (*a*) What is this common potential difference? (*b*) What is the energy stored in the electric field before and after the switch S in Fig. 30-11 is closed?

Solution (*a*) Electric charge must be conserved, so the original charge q_0 is shared by two capacitors, or

$$q_0 = q_1 + q_2.$$

Applying the relation $q = C\,\Delta V$ to each term yields

$$C_1\,\Delta V_0 = C_1\,\Delta V + C_2\,\Delta V.$$

or

$$\Delta V = \Delta V_0 \frac{C_1}{C_1 + C_2} = \frac{(6.30\text{ V})(3.55\ \mu\text{F})}{3.55\ \mu\text{F} + 8.95\ \mu\text{F}} = 1.79\text{ V}.$$

If we know the battery voltage ΔV_0 and the value of C_1, we can determine an unknown capacitance C_2 by measuring the value of ΔV in an arrangement similar to that of Fig. 30-11.

(*b*) The initial stored energy is

$$U_i = \tfrac{1}{2}C_1(\Delta V_0)^2 = \tfrac{1}{2}(3.55 \times 10^{-6}\text{ F})(6.30\text{ V})^2$$
$$= 7.05 \times 10^{-5}\text{ J} = 70.5\ \mu\text{J}.$$

The final energy is

$$U_f = \tfrac{1}{2}C_1(\Delta V)^2 + \tfrac{1}{2}C_2(\Delta V)^2 = \tfrac{1}{2}(C_1 + C_2)(\Delta V)^2$$
$$= \tfrac{1}{2}(3.55 \times 10^{-6}\text{ F} + 8.95 \times 10^{-6}\text{ F})(1.79\text{ V})^2$$
$$= 2.00 \times 10^{-5}\text{ J} = 20.0\ \mu\text{J}.$$

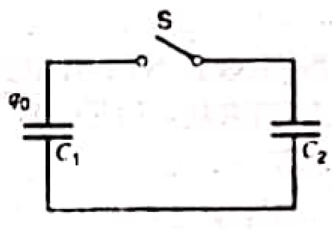

FIGURE 30-11. Sample Problem 30-6. Capacitor C_1 has previously been charged to a potential difference ΔV_0 by a battery that has been removed. When the switch S is closed, the initial charge q_0 on C_1 is shared with C_2.

energy in the connecting wires, as we discuss in the next chapter.*

SAMPLE PROBLEM 30-7. An isolated conducting sphere whose radius R is 6.85 cm carries a charge q = 1.25 nC. (*a*) How much energy is stored in the electric field of this charged conductor? (*b*) What is the energy density at the surface of the sphere? (*c*) What is the radius R_0 of an imaginary spherical surface such that one-half of the stored potential energy lies within it?

Solution (*a*) From Eqs. 30-24 and 30-12 we have

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\epsilon_0 R} = \frac{(1.25 \times 10^{-9}\ \text{C})^2}{(8\pi)(8.85 \times 10^{-12}\ \text{F/m})(0.0685\ \text{m})}$$

$$= 1.03 \times 10^{-7}\ \text{J} = 103\ \text{nJ}.$$

(*b*) To find the energy density, we must first find E at the surface of the sphere. This is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}.$$

The energy density is then, using Eq. 30-28,

$$u = \tfrac{1}{2}\epsilon_0 E^2 = \frac{q^2}{32\pi^2\epsilon_0 R^4}$$

$$= \frac{(1.25 \times 10^{-9}\ \text{C})^2}{(32\pi^2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0685\ \text{m})^4}$$

$$= 2.54 \times 10^{-5}\ \text{J/m}^3 = 25.4\ \mu\text{J/m}^3.$$

(*c*) The energy that lies in a spherical shell between radii r and $r + dr$ is

$$dU = (u)(4\pi r^2)(dr),$$

where $(4\pi r^2)(dr)$ is the volume of the spherical shell. Using the result of part (*b*) for the energy density evaluated at a radius r, we obtain

$$dU = \frac{q^2}{32\pi^2\epsilon_0 r^4}4\pi r^2\, dr = \frac{q^2}{8\pi\epsilon_0}\frac{dr}{r^2}.$$

The condition given for this problem is

$$\int_R^{R_0} dU = \frac{1}{2}\int_R^{\infty} dU$$

or, using the result obtained above for dU and canceling constant factors from both sides,

$$\int_R^{R_0}\frac{dr}{r^2} = \frac{1}{2}\int_R^{\infty}\frac{dr}{r^2},$$

which becomes

$$\frac{1}{R} - \frac{1}{R_0} = \frac{1}{2R}.$$

Solving for R_0 yields

$$R_0 = 2R = (2)(6.85\ \text{cm}) = 13.7\ \text{cm}.$$

*Some slight amount of energy is also radiated away. For a critical discussion, see "Two-Capacitor Problem: A More Realistic View," by R. A. Powell, *American Journal of Physics*, May 1979, p. 460.

30-6 CAPACITOR WITH DIELECTRIC

In Section 29-6 we discussed the effect of applying an electric field to an insulating material (a dielectric). We showed that the effect of the dielectric is to reduce the strength of the electric field in its interior from its initial value E_0 in vacuum to $E = E_0/\kappa_e$ inside the dielectric. The parameter κ_e, the dielectric constant, has values greater than 1 for all materials, so that the electric field in the dielectric is smaller than the field in vacuum.

In this section, we consider the effect of filling the interior of a capacitor with a dielectric material. This effect was first investigated in 1837 by Michael Faraday. Faraday constructed two identical capacitors, filling one with a dielectric material and leaving the other with air between its plates. When both capacitors were connected to batteries with the *same potential difference,* Faraday found that the charge on the capacitor filled with the dielectric was greater than the charge on the capacitor with air between its plates. That is, the presence of the dielectric enables the capacitor to store more charge. Since storage of charge for later discharge is one of the purposes for which we use capacitors, the presence of a dielectric can enhance the performance of a capacitor.

The effect of filling a capacitor with dielectric depends on whether we do so with the battery connected (as in Faraday's experiment) or disconnected. First we consider the situation as in Faraday's experiment (Fig. 30-12). A capacitor with capacitance C is connected to a battery of potential difference ΔV and allowed to become fully charged, such that the plates carry a charge q, as in Fig. 30-12*a*. With the battery remaining connected, we then fill the interior of the capacitor with a material of dielectric constant κ_e, as in Fig. 30-12*b*. The battery maintains the same potential difference ΔV across the plates.

Equation 30-2 shows that, if the potential differences in Figs. 30-12*a* and 30-12*b* are the same, then the electric fields inside the capacitor must be the same. However, we

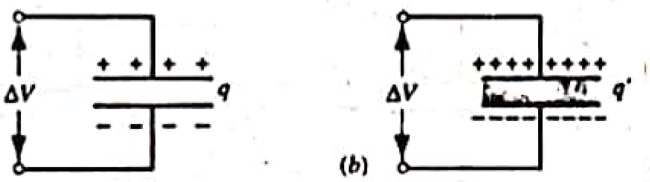

FIGURE 30-12. (*a*) An empty capacitor is charged by connecting it to a battery that establishes a potential difference ΔV. (*b*) The battery remains connected as the capacitor is filled with a dielectric. In this case, the potential difference ΔV remains constant, but q increases.