5 Angle measurement

As shown in Chapter 1, horizontal and vertical angles are fundamental measurements in surveying.

The vertical angle, as already illustrated, is used in obtaining the elevation of points (trig levelling) and in the reduction of slant distance to the horizontal.

The horizontal angle is used primarily to obtain direction to a survey control point, or to topographic detail points, or to points to be set out.

An instrument used for the measurement of angles is called a theodolite, the horizontal and vertical circles of which can be likened to circular protractors set in horizontal and vertical planes. It follows that, although the points observed are at different elevations, it is always the horizontal angle and not the space angle which is measured. For example, observations to points A and C from B (*Figure 5.1*) will give the horizontal angle $ABC = \theta$. The vertical angle of elevation to A is α and its zenith angle is Z_A.

5.1 THE THEODOLITE

There are basically two types of theodolite, the optical mechanical type or the electronic digital type, both of which may be capable of reading directly to $1'$, $20''$, $1''$ or $0.1''$ of arc, depending upon the precision of the instrument. The selection of an instrument specific to the survey tolerances of the work in hand is usually overridden by the commercial considerations of the company and a $1''$ instrument may be used for all work. When one considers that $1''$ of arc subtends 1 mm in 200 m, it is sufficiently accurate for practically all work carried out in engineering.

Figure 5.2 shows a typical theodolite, whilst *Figure 5.3* shows the main components of a theodolite. This exploded diagram enables the relationships of the various parts to be more clearly understood along with the relationships of the main axes. In a correctly adjusted instrument these axes should all be normal to each other, with their point of intersection being the point about which the angles are measured. Neither figure illustrates the complexity of a modern theodolite or the very high calibre of the process of its production.

The basic features of a typical theodolite are, with reference to *Figure 5.3*, as follows:

(1) The trivet stage forming the base of the instrument connects it to the tripod head.
(2) The tribrach supports the rest of the instrument and with reference to the plate bubble can be levelled using the footscrews which act against the fixed trivet stage.
(3) The lower plate carries the horizontal circle which is made of glass, with graduations from 0° to 360° photographically etched around the perimeter. This process enables lines of only 0.004 mm thickness to be sharply defined on a small-diameter circle (100 mm), thereby resulting in very compact instruments.
(4) The upper plate carries the horizontal circle index and fits concentrically with the lower plate.
(5) The plate bubble is attached to the upper plate and when adjusted, using the footscrews, makes the instrument axis vertical. Some modern digital or electronic theodolites have replaced the spirit bubble with an electronic bubble.

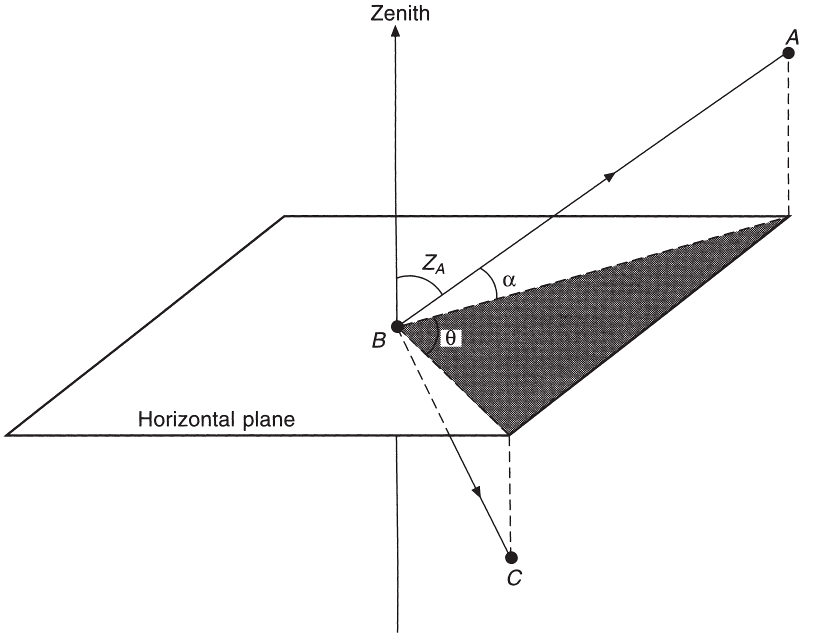


Fig. 5.1 *Horizontal, vertical and zenith angles*

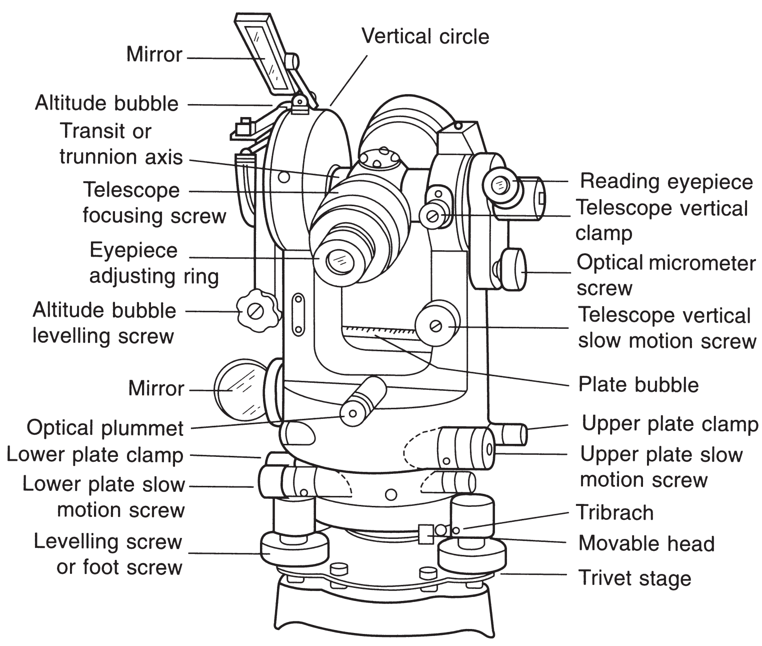


Fig. 5.2 *Typical optical mechanical theodolite*

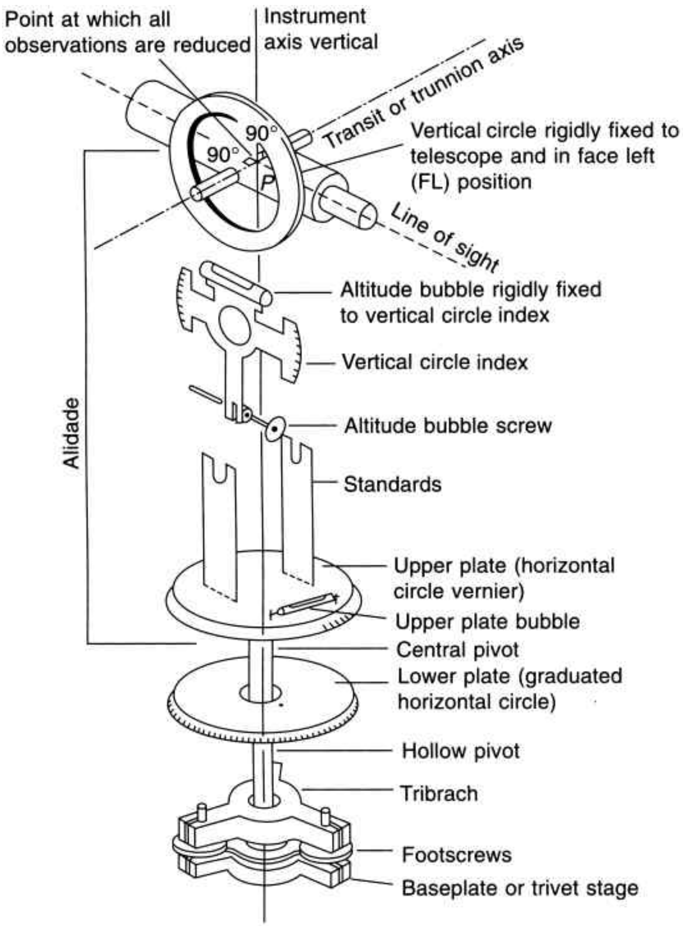


Fig. 5.3 *Simplified theodolite*

(6) The upper plate also carries the standards which support the telescope by means of its transit axis. The standards are tall enough to allow the telescope to be fully rotated about its transit axis.
(7) The vertical circle, similar in construction to the horizontal circle, is fixed to the telescope axis and rotates with the telescope.
(8) The vertical circle index, against which the vertical angles are measured, is set normal to gravity by means of (a) an altitude bubble attached to it, or (b) an automatic compensator. The latter method is now universally employed in modern theodolites.
(9) The lower plate clamp (*Figure 5.2*) enables the horizontal circle to be clamped into a fixed position. The lower plate slow motion screw permits slow movement of the theodolite around its vertical axis, when the lower plate clamp is clamped. Most modern theodolites have replaced the lower plate clamp and slow motion screw with a horizontal circle-setting screw. This single screw rotates the horizontal circle to any reading required.
(10) Similarly, the upper plate clamp and slow motion screw have the same effect on the horizontal circle index.
(11) The telescope clamp and slow motion screw fix and allow fine movement of the telescope in the vertical plane.

(12) The altitude bubble screw centres the altitude bubble, which, as it is attached to the vertical circle index, makes it horizontal prior to reading the vertical circle. As stated in (8), this is now done by means of an automatic compensator.
(13) The optical plummet, built into either the base of the instrument or the tribrach (*Figure 5.12*), enables the instrument to be centred precisely over the survey point. The line of sight through the plummet is coincidental with the vertical axis of the instrument.
(14) The telescopes are similar to those of the optical level but usually shorter in length. They also possess rifle sights or collimators for initial pointing.

5.1.1 Reading systems

The theodolite circles are generally read by means of a small auxiliary reading telescope at the side of the main telescope (*Figure 5.2*). Small circular mirrors reflect light into the complex system of lenses and prisms used to read the circles.

There are basically three types of reading system; optical scale reading, optical micrometer reading and electronic digital display.

(1) The optical scale reading system is generally used on theodolites with a resolution of 20″ or less. Both horizontal and vertical scales are simultaneously displayed and are read directly with the aid of the auxiliary telescope.

The telescope used to give the direct reading may be a 'line microscope' or a 'scale microscope'.

The line microscope uses a fine line etched onto the graticule as an index against which to read the circle.

The scale microscope has a scale in its image plane, whose length corresponds to the line separation of the graduated circle. *Figure 5.4* illustrates this type of reading system and shows the scale from 0′ to 60′ equal in scale of one degree on the circle. This type of instrument is frequently referred to as a direct-reading theodolite and, at best, can be read, by estimation, to 20″.

(2) The optical micrometer system generally uses a line microscope, combined with an optical micrometer using exactly the same principle as the parallel plate micrometer on a precise level.

Figure 5.5 illustrates the principle involved. If the observer's line of sight passes at 90° through the parallel plate glass, the circle reading would be 23° 20′ + *S*, with the value of *S* unknown. The parallel plate is rotated using the optical micrometer screw (*Figure 5.2*) until the line of sight is at an exact reading of 23° 20′ on the circle. This is as a result of the line of sight being refracted towards the normal and emerging on a parallel path. The distance *S* through which the observer's line of sight was displaced is recorded on the micrometer scale as 11′ 40″.

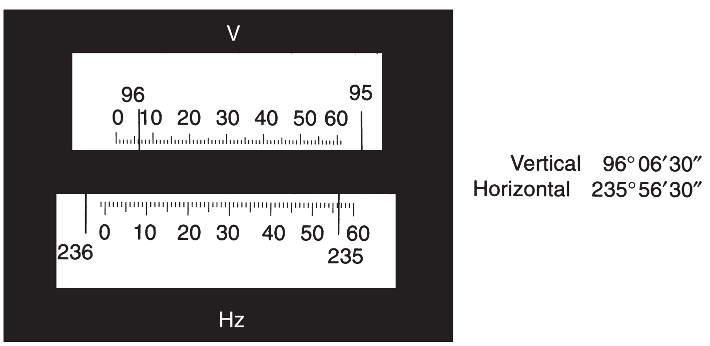


Fig. 5.4 *Wild T16 direct reading theodolite*

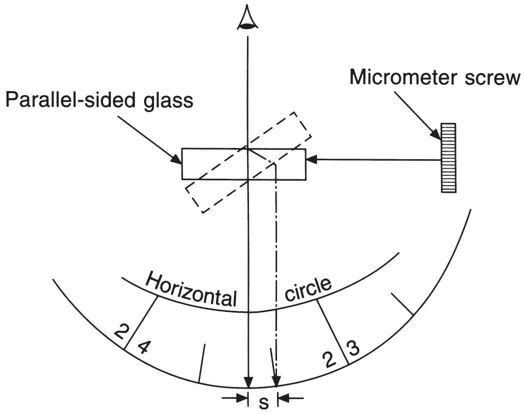


Fig. 5.5 *Micrometer*

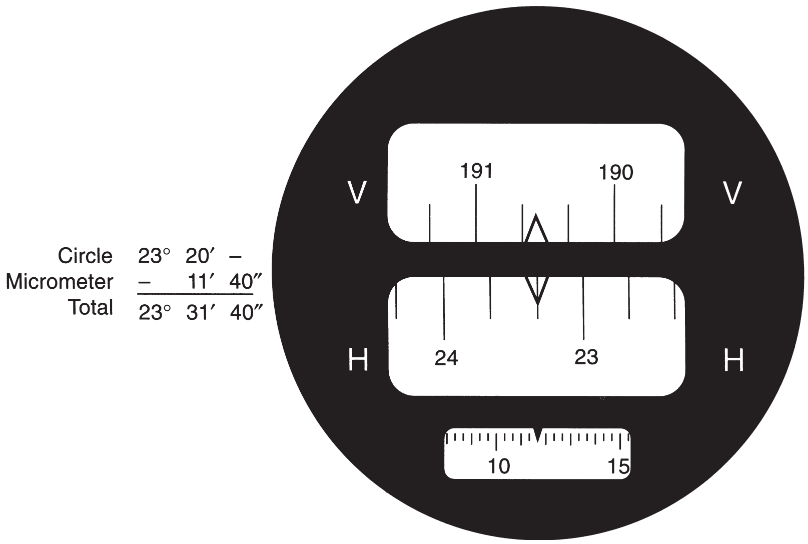
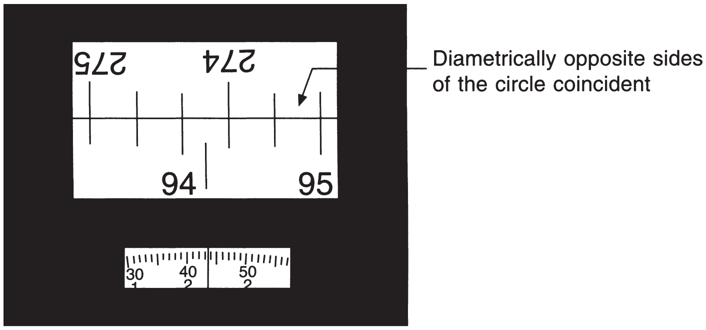


Fig. 5.6 *Watts Microptic No 1 theodolite reading system*

The shift of the image is proportional to the angle of tilt of the parallel plate and is read on the micrometer scale. Before the scale can be read, the micrometer must be set to give an exact reading (23° 20′), as shown on *Figure 5.6*, and the micrometer scale reading (11′ 40″) added on. Thus the total reading is 23° 31′ 40″. In this instance the optical micrometer reads only one side of the horizontal circle, which is common to 20″ instruments.

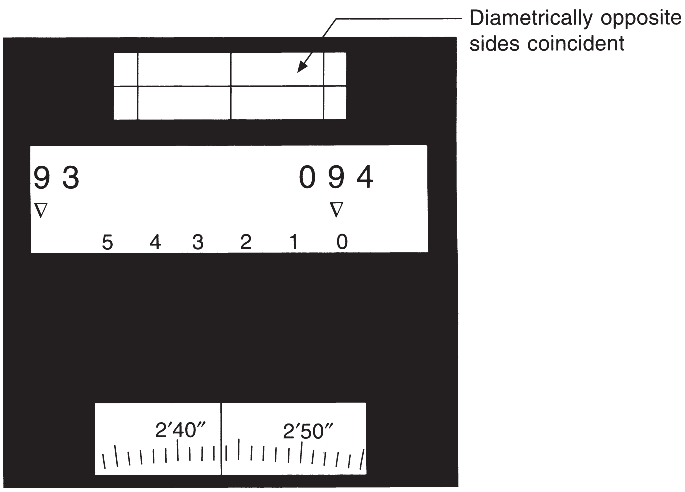
On more precise theodolites, reading to 1″ of arc and above, a coincidence microscope is used. This enables diametrically opposite sides of the circle to be combined and a single mean reading taken. This mean reading is therefore free from circle eccentricity error.

Figure 5.7 shows the diametrically opposite scales brought into coincidence by means of the optical micrometer screw. The number of divisions on the main scale between 94° and 95° is three; therefore each division represents 20′. The indicator mark can only take up one of two positions, either mid-division or on a full division. In this case it is mid-division and represents a reading of 94° 10′;



Circle	94° 10′ –
Micrometer	– 2′ 44″
Total	94° 12′ 44″ (to nearest 1″)

Fig. 5.7 *Wild T2 (old pattern) theodolite reading system*



Circle	94° 10′ –
Micrometer	– 2′ 44″
Total	94° 12′ 44″ (to nearest 1″)

Fig. 5.8 *Wild T2 (new pattern)*

the micrometer scale reads 2′ 44″ to the nearest second, giving a total reading of 94° 12′ 44″. An improved version of this instrument is shown in *Figure 5.8*.

The above process is achieved using two parallel plates rotating in opposite directions, until the diametrically opposite sides of the circle coincide.

(3) There are basically two systems used in the electro-optical scanning process, either the incremental method or the code method (*Figure 5.9*).

The basic concept of the incremental method can be illustrated by considering a glass circle of 70–100 mm diameter, graduated into a series of radial lines. The width of these photographically etched lines is equal to their spacing. The circle is illuminated by a light diode; a photodiode, equal in width to a graduation, forms an index mark. As the alidade of the instrument rotates, the glass circle moves in relation to the diode. The light intensity signal radiated approximates to a sine curve.

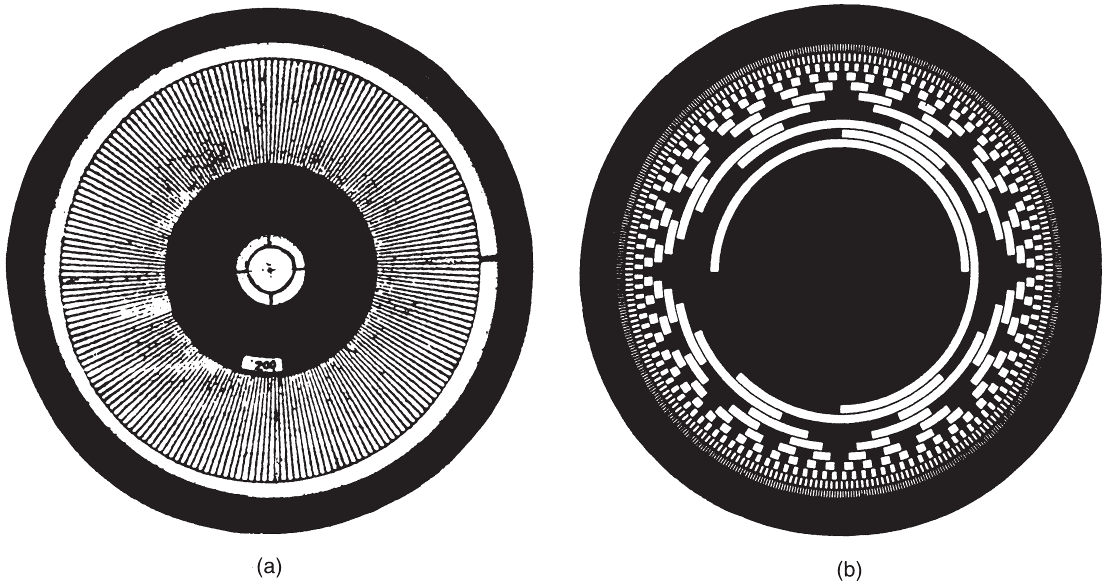


Fig. 5.9 *(a) Incremental disk, (b) binary coded disk*

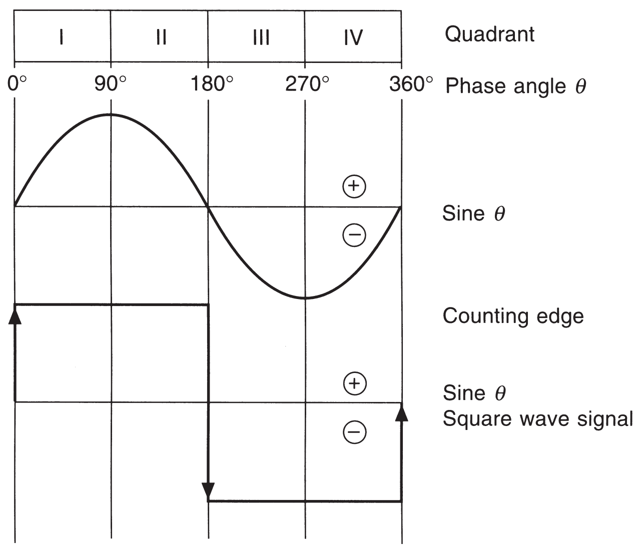


Fig. 5.10 *Sine wave to square wave modulation*

The diode converts this to an electrical signal correspondingly modulated to a square wave signal (*Figure 5.10*). The number of signal periods is counted by means of the leading and trailing edges of the square wave signal and illustrated digitally in degrees, minutes and seconds on the LCD. This simplified arrangement would produce a relatively coarse least count resolution, requiring further refinement.

For example, consider a glass circle that contains 20 000 radial marks, each 5.5 μm thick, with equal width spacing. A section of the circle comprising 200 marks is superimposed on the diametrically

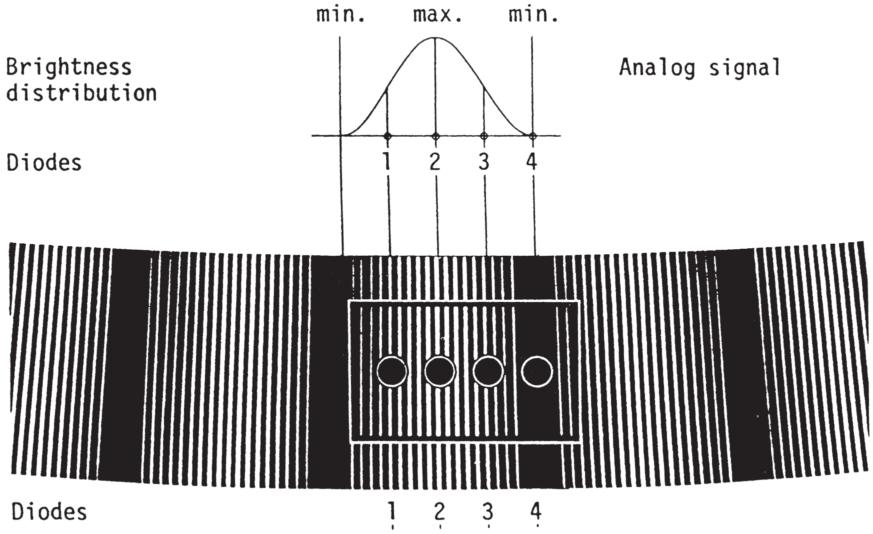


Fig. 5.11 *Fine reading using moiré pattern brightness*

opposite section, forming a moiré pattern. A full period (light–dark variation) corresponds to an angular value of approximately 1 min of arc, with a physical length of 2 mm. A magnification of this period by two provides a length of 4 mm over which the brightness pattern can be electronically scanned. Thus the coarse measurement can be obtained from 40 000 periods per full circle, equivalent to 30″ per period.

The fine reading to 0.3″ is obtained by monitoring the brightness distribution of the moiré pattern using the four diodes shown (*Figure 5.11*). The fine measurement obtains the scanning position location with respect to the leading edge of the square wave form within the last moiré pattern. It is analogous to measuring the fraction of a wavelength using the phase angle in EDM measurement.

The code methods use coded graduated circles (*Figure 5.9*(*b*)). Luminescent diodes above the glass circle and photodiodes below, one per track, detect the light pattern emitted, depending on whether a transparent track (signal 1) or an opaque track (signal 0) is opposite the diode at that instant. The signal is transferred to the computer for processing into a digital display. If there are n tracks, the full circle is divided into 2^n equal sectors. Thus a 16-track disk has an angular resolution of 2^{16}, which is 65 532 parts of a full circle and is equivalent to a 20″ resolution.

The advantage of the electronic systems over the glass arc scales is that they produce a digital output free from misreading errors and in a form suitable for automatic data recording and processing. *Figure 5.12* illustrates the glass arc and electronic theodolites.

5.2 INSTRUMENTAL ERRORS

In order to achieve reliable measurement of the horizontal and vertical angles, one must use an instrument that has been properly adjusted and adopt the correct field procedure.

In a properly adjusted instrument, the following geometrical relationships should be maintained (*Figure 5.3*):

(1) The plane of the horizontal circle should be normal to the vertical axis of rotation.
(2) The plane of the vertical circle should be normal to the horizontal transit axis.

(a) (b)

Fig. 5.12 *(a) Wild T1 glass arc theodolite with optical plummet in the alidade, (b) Wild T1600 electronic theodolite with optical plummet in the tribrach*

(3) The vertical axis of rotation should pass through the point from which the graduations of the horizontal circle radiate.
(4) The transit axis of rotation should pass through the point from which the graduations of the vertical circle radiate.
(5) The principal tangent to the plate bubble should be normal to the main axis of rotation.
(6) The line of sight should be normal to the transit axis.
(7) The transit axis should be normal to the main axis of rotation.
(8) When the telescope is horizontal, the vertical circle indices should be horizontal and reading zero, and the principal tangent of the altitude bubble should, at the same instance, be horizontal.
(9) The main axis of rotation should meet the transit axis at the same point as the line of sight meets this axis.
(10) The line of sight should maintain the same position with change of focus (an important fact when coplaning wires).

Items (1), (2), (3) and (4) above are virtually achieved by the instrument manufacturer and no provision is made for their adjustment. Similarly, (9) and (10) are dealt with, as accurately as possible, in the manufacturing process and in any event are minimized by double face observations. Items (5), (6), (7) and (8) can, of course, be achieved by the usual adjustment procedures carried out by the operator.

The procedure referred to above as ‘double face observation’ is fundamental to the accurate measurement of angles. An examination of *Figure 5.2* shows that an observer looking through the eyepiece of the telescope would have the vertical circle on the left-hand side of his/her face; this would be termed a ‘face left’ (FL) observation. If the telescope is now rotated through 180° about its transit axis and then the instrument rotated through 180° about its vertical axis, the vertical circle would be on the right-hand side of the observer’s face when looking through the telescope eyepiece. This is called a ‘face right’ (FR) observation.