THE EFFICIENT ALLOCATION OF RESOURCES IN EDUCATION *

Samuel Bowles

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I. Introduction

Recognition of the important contribution of education to economic growth has heightened the interest of economists and economic planners in the development of an economically rational basis for the allocation of resources in the educational sector. A number of recent models of the entire economy have explicitly incorporated inputs of labor of various skill or educational levels.[1] In addition, economists have directed their attention toward the educational sector itself and have attempted to develop methods which yield economically rational patterns of resource allocation and enrollments within the educational system.[2]

The model described below is addressed to four major questions concerning the efficiency of the educational system as a producer of educated labor, namely:

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1. For, example, Michael Bruno, "Experiments with a Multi-Sectoral Programming Model," in Irma Adelman and E. Thorbecke, *Theory and Design of Economic Development* (Baltimore: Johns Hopkins Press, 1966), and Bruno's article with the same title in *Review of Economics and Statistics*, forthcoming. Also see the treatment of the labor inputs in the model of the Cambridge Growth Project, as reported in Alan Brown, *et al.*, "Output Manpower and Industrial Skills in the United Kingdom," in Organisation for Economic Cooperation and Development, Study Group in the Economics of Education, *The Residual Factor and Economic Growth* (Paris, 1964), pp. 240–63.

2. See Richard Stone, "A Model of the Educational System," *Minerva*, III (Winter 1965), 172–87.

1. What amount of society's resources should be devoted to education?

2. How should the total resource use be distributed among various types of education?

3. What educational technologies should be chosen?

4. What is the optimal level and composition of the importation of labor for use within the educational system?

This model differs from most existing approaches to educational planning in the following ways:

1. It is based on the principle of constrained maximization and involves the explicit consideration of both the costs and benefits of various educational programs.

2. Use of the model allows the simultaneous computation of optimal enrollment levels in each type of education, an optimal pattern of importation (or exportation) of educated labor, and the choice of efficient educational technologies.

3. The model is based on the assumption that each category of educated labor is highly substitutable both vis à vis other types of labor and vis à vis capital. In this respect the model differs significantly from most other planning approaches, which assume that the production functions in the economy are characterized by fixed input coefficients for labor classified by occupational group or educational level.

4. It deals directly with labor classified by educational level.[3] This feature of the model avoids the problem of translating demands for labor classified by occupational group into demands for the outputs of specific educational levels.[4]

Although this paper is devoted primarily to a discussion of the model, a number of observations on its application to Northern Nigeria will be made.[5] Section II of this paper contains a brief outline of the model and a sketch of the structure of the educational system of Northern Nigeria. Sections III and IV present the objective function and the constraint equations, respectively, along with

3. In this respect it is similar to the model presented by Jan Tinbergen and Hector Correa, "Quantitative Adaptation of Education to Accelerated Growth," *Kyklos*, XV (1962) and the more recent versions of the original model.

4. The conversion of occupational into educational classifications is generally accomplished on the basis of the concept of an educational "requirement" (or a distribution of "requirements") for the average performance of each occupation. Cf. Richard S. Eckaus, "Economic Criteria for Education and Training," *Review of Economics and Statistics*, XLVI (May 1964).

5. A detailed description of the model and its application to Northern Nigeria has been presented in Samuel Bowles, "The Efficient Allocation of Resources in Education: A Planning Model with Applications to Northern Nigeria," unpublished Ph.D. thesis, Harvard University, 1965.

some related data. Sections V through VIII contain a discussion of the application of the model to actual policy problems.

Some of the more important results based on the operation of the model with Nigeria data may be summarized as follows:

1. The educational sector has an extremely strong claim on economic resources.

2. Efficient allocation of resources within the educational system requires a rapid expansion of primary education and a reduction in enrollments in technical and secondary schools.

3. The introduction of new educational technologies allows for major increases in the efficiency of the system.

4. The productivity of foreigners imported to teach in the system is very high at the present levels of importation.

The optimal enrollments in various types of schools based on solutions of this model appear to differ considerably from Nigerian educational plans based on the manpower requirements approach.

II. An Outline of the Model

We seek to maximize a weighted function of enrollments in various types of educational institutions over time, subject to constraints based on an educational production technology and given resource availabilities. The constraint equations define what can be called an intertemporal production possibility set for the educational system. The objective function is the contribution of the educational system to future national income, measured by the increment in discounted lifetime earnings attributable to additional years of education.

The educational system is represented in this model as an aggregation of production activities. In the application of the model to Northern Nigeria, the educational activities included primary education, secondary education, higher education, various types of teacher training, and technical and vocational education.[6] Each of these processes used a variety of inputs (both human and otherwise) to transform raw materials (the uneducated) or intermediate goods (continuing students) into a producer's good.[7] Relationships between educational activities are presented as a system of intertemporal flows of students and teachers. The output of a given educational institution can be allocated to one of three tasks:

1. continuation of his education at a higher level,

6. Cf. below, Table I.
7. The system concurrently produces a consumer's good, "education."

2. employment as a teacher at a lower level, and

3. employment in the labor force outside the educational system.[8]

The structure of flows of teachers and students within the educational system is best described in the usual input-output format, as in Table I, which presents the intra-educational flows among the nine major types of formal education in Northern Nigeria.[9]

The constraints relate to the use by the educational system of inputs supplied from outside the educational system, (e.g., expenditure on education, total population in the school-going age group), as well as endogenously produced inputs (teachers of various types, student outputs from one educational process who appear as inputs into higher educational processes). In addition, boundary conditions limit the policy instruments to values which are judged to be politically and administratively feasible.

The method described here is a sectoral model of the educational system. Production processes in the rest of the economy are not included explicitly. Thus the demand functions for the outputs of the educational system and the supply functions for the exogenously supplied educational inputs are specified prior to the operation of the model.

The instrument variables in the model include enrollments and resource use at the various educational levels, and additional instruments which require discontinuous or institutional changes. Examples of the latter are choices involving new educational technologies (e.g., increased use of audio-visual equipment) or changes in the structure of the system (e.g., extending university education to a four year course). The instrument variables have been defined so as to correspond to the actual policy instruments available to most governments. In addition to the instrument variables relating to the production of specific types of education, the system is allowed to import a number of types of educated labor, and to send students abroad for their education. Thus for some types of labor, the system is presented with a three-way choice: the production of labor with a given level of educational attainment either within the country or in foreign educational institutions, or the importation of

8. Some of the outputs will either not seek employment, or will for some period of time be involuntarily unemployed.

9. Note that the table has been arranged so that all of the flows of students (intermediate goods) lie above the diagonal, while flows of teachers (capital goods) lie below the diagonal.

Lack of data prevented the inclusion of various educational activities outside of the formal educational system, e.g., on-the-job training and adult education.

TABLE I

THE EDUCATIONAL SYSTEM OF NORTHERN NIGERIA[1]

Producing Sectors	Usual Age of Entry	Duration of Course in Years	1	2	3	4	5	6	7	8	9	Labor Market
1. Primary School	6	7		S	S	S						L
2. Craft School	13	3						S				
3. Grade III Teacher's Course (Teacher Training College)	13	3	T	T			S					
4. Secondary School	13	5							S	S		L
5. Grade II Teacher's Course (Teacher Training College)	16	2	T	T	T	T	T			S		
6. Technical Training School	16	3										L
7. Form VI	18+	2									S	L
8. Northern Secondary Teacher's College (N.S.T.C.)	18+	3	T	T	T	T	T	T				
9. University Study	18+	3			T	T	T	T	T	T		L

Note: T indicates a flow of teachers; S indicates a flow of students; L indicates a flow of school-leavers to the labor market

1. Some insignificant flows which have not been represented in the model have been excluded from this table.

foreign labor possessing the educational attainments in question. Additional activities allow the system to recruit back into the educational sector personnel trained as teachers but who are presently working in nonteaching positions.

The model encompasses a number of time periods, so as to allow consideration of the intertemporal relationships within the educational system. Educational decisions involving enrollments, resource use, and hiring of staff are generally incorporated in annual budgets or similar documents, and are made prior to the beginning of the school year to be implemented in the course of the year. It is thus appropriate to select the year as the time unit used in the model.

In actual application the model should probably be operated on a year-by-year sequential basis. If the planning period is n years, the model can be operated in year 0 (the base year) and the results for the years $1 \ldots n$ computed. Only the enrollments and resource allocation for the year 1 must be acted on at that time, so that at the end of year 1 the model can be operated once more, incorporating new information on either the production processes or the present values of the educational output. The results for years $2 \ldots n + 1$ can then be calculated, the values of the instrument variables for year 2 acted upon, and the process continued.[1]

Solutions of the model yield optimal values of the instrument variables in each year of the planning period, namely:

1. a time pattern of enrollments and resource use in each type of education;

2. levels of recruitment of new inputs (e.g., foreign teachers and domestic ex-teachers) to the system;

3. an efficient choice of educational techniques including such choices as foreign as opposed to domestic university study. The solutions also generate shadow prices for resources used in the production of education.

While the values of the instrument variables for any given solution are interesting in themselves, results to be gained through parametrically programming some of the crucial elements in the model are probably more useful from the standpoint of policy-making. The model not only allows us to explore the production pos-

1. Operation of the model in this manner is probably a good reflection of the actual policy-making process, which proceeds from year to year rather than on a once-for-all basis for an entire n-year period. In addition it allows the efficient use of new data. A further advantage is that it avoids the necessity of acting on the values of the instrument variables in the later years in the planning period, which are presumably sensitive to the somewhat arbitrary terminal conditions.

sibility set for the educational system, but also to measure the trade-off between the availability of particular inputs, on the one hand, and the values of the instrument variables, the objective function, and the shadow prices on the other.

III. The Objective Function

The objective function used in this model represents the net economic benefits associated with the educational activities, namely, the present value of the economic benefits associated with the output of all levels of the educational system over a number of years minus the present value of the associated costs.

As the social welfare function presumably contains many components which have some functional relation to education, it is useful to distinguish between those educational benefits which operate via the income or income-related terms of the welfare function, and those which operate on other components.

We will call the former "economic" and the latter "noneconomic," although any dichotomous distinction of this type is bound to be somewhat arbitrary. This classification excludes from the category of "economic" benefits those consequences of education generally called "consumption benefits," namely those which accrue to the student in the form of pleasure in studying or later in being an educated man and having access to the style of life open to those with education.

Any consequence of the educational system's output which results in an increase in the value of present or future national income is thus defined as an economic benefit of education. If we confine attention to the level of income rather than its distribution the maximization of net economic benefits corresponds to the maximization of the contribution of the educational system to the future (discounted) national income.[2]

Ideally, we would like to measure the economic benefits by the increase in an individual's social marginal productivity resulting

2. The exclusion of the noneconomic benefits is not intended to suggest that these should be ignored in the construction of the educational plan. The economically efficient patterns of allocation yielded by the model presented here are intended to be one input into the planning process, in competition with other allocation plans based on noneconomic considerations. The function of this approach is not to specify one socially desirable pattern of allocation, but rather to clarify the economic benefits and costs of the educational choices facing a society. An alternative approach, based on a simple hypothetical planner's preference function has been used in the application of a similar model (Samuel Bowles, "A Planning Model for the Efficient Resources in Education," May 1964, mimeographed).

from his education. The social marginal productivity of an educational output can be described as the total effect on future national income attributable to the individual's education, taking into account his direct contribution to output, as well as any external effects which may exist.[3] In the application of this model to the educational planning problems of Northern Nigeria, earnings were used as an estimate of the marginal productivity of each category of labor. While this measure is subject to a number of objections, it was thought to be a rough indication of the private marginal productivity of the worker.

In view of the fact that each educational output has a working life extending over a number of time periods, future increases in labor productivity generated by the educational system are discounted at an appropriate rate of time preference.

The direct social costs associated with each activity are the present value of the annual per student costs summed over the duration of the educational course. The cost of one student year is the sum of the required inputs valued at their opportunity cost, that is, their social marginal productivity in their next best use, or at their social marginal cost.[4] The cost of education to the educational institution is not the relevant cost figure, as it includes items of private as well as social cost, such as feeding the students and perhaps housing and clothing them, services which if not undertaken at the school would have to be undertaken in the home.[5]

The indirect cost element relates to the withdrawal of students from the labor force (or their retention in the educational system) for the continuation of the education. Students' time should be valued at its opportunity cost, namely, the social marginal productivity of the student if he were on the labor market. Measure-

3. On the external effects see Burton Weisbrod, *The External Benefits of Public Education, an Economic Analysis* (Princeton: Princeton University Industrial Relations Section, 1964).

4. While the relevant cost concept is marginal rather than average cost, in most educational activities studied in Nigeria there were good grounds for assuming that the two quantities coincided. The expansion of primary education, for example, requires a nearly proportional duplication of the existing processes through the addition of production units (schools) of the same scale and input structure as those presently existing. In the field of university education, however, there are significant indivisibilities and fixed costs, and consequently a major divergence between average and marginal costs. In the case of university education it was judged likely that additional enrollments would be accommodated in existing institutions with less than proportional changes in existing plant and equipment. In these cases marginal costs (over the relevant range) were estimated and used in the operation of the model.

5. Naturally, if the marginal cost of these services when provided by the school differs from their marginal cost when provided at home, the difference (positive or negative) should be attributed to education.

ment of the social marginal productivity of the student must include consideration of his prospects for being employed were he to leave school.

The net benefits coefficient associated with each activity is the present value of the estimated stream of lifetime earnings corresponding to the type of labor produced, (Y_j), minus the present value of the foregone stream of lifetime earnings corresponding to the type of labor used as a student input into the production process, $(Y_{j'})$, and minus also the present value of the direct costs, (C_j). Thus net benefits for education j are

(3.1) $$Z_j = Y_j - Y_{j'} - C_j$$

and, using the p superscript to indicate the year of the planning period in which a student is admitted to the given level j, we may define the objective function as

(3.2) $$Z^* = \sum_j \sum_p Z^p{}_j X_j{}^p.$$

The earnings data were based on a sample survey of employ-

TABLE II

THE PRESENT VALUE OF THE NET BENEFITS ASSOCIATED WITH VARIOUS EDUCATIONAL ACTIVITIES IN 1964[1]

(adjusted for wastage, failures, labor force participation and unemployment)

ACTIVITY (1)	Present Value of Lifetime Earnings Y_j (2)	Present Value of Lifetime Earnings Foregone[2] $Y_{j'}$ (3)	Increment in Present Value of Earnings (2)–(3) (4)	Present Value of Direct Social Costs C_j (5)	Present Value of Net Benefits (2–3–5) Z_j (6)	Ratio of Present Values of Increment in Earnings to Direct Costs[3] (4)/(5) (7)
Primary School	1659	611	1048	62	986	16.9
Secondary School	4592	2910	1682	476	1206	3.5
Technical Training School	4337	2713	1624	785	839	2.1
Form VI	7460	7356	104	326	−222	0.3
University Studies	20559	9130	11429	1350	10079	8.5
University Studies Abroad	20559	9130	11429	1730	9699	6.6

Source: See text. The basic data are reported in Samuel Bowles, "The Efficient Allocation of Resources in Education," *op. cit.*, Chaps. 5 and 6.

Note: All figures are in pounds and are based on a 5 per cent discount rate.

1. Net benefits coefficients for activities making no direct deliveries to the labor market (i.e., craft school, which serves as a feeder for technical training school plus the three types of teacher training) do not appear in this table. The demand for the outputs of these activities is derived endogenously from the admissions levels in the optimal solution. The objective function coefficients for these activities are based on the direct costs plus earnings foregone during the process of education. The net benefits coefficients in this table refer only to activities in the base year, 1964.

2. The present value of income foregone is the discounted lifetime earnings of an individual who enters the labor force with the prerequisites for admission to level j. Thus the alternative earnings stream from Form VI is the stream accruing to those who had passed the West African School Certificate, not the composite secondary school stream adjusted for failures, dropouts, etc.

3. The ratios in Column 7 are not used in the operation of the model. They are presented here merely for reference.

ment in private firms in Northern Nigeria in 1965. Costs were estimated on the basis of school by school financial records with expenditures grouped in a number of functional categories. Standard architectural plans and associated cost data were used to estimate the annual capital costs. The resulting net benefits coefficients and some of the underlying data are presented in Table II.

IV. The Constraints

The education production technology is represented by a set of fixed input coefficients production functions. The choice of educational production functions embodying fixed input coefficients is justified on the grounds that while a considerable amount of input substitution may in fact be possible from a pedagogical standpoint, many educational administrators appear to believe that at any given time the appropriate teacher-student ratios and other input coefficients are roughly fixed, and insist on a common educational process in all schools of the same type.[6]

For any level of education, j, in period P the production function can be written:

$$(4.1) \qquad X^p_j = \min_{i,\,t} \left[\frac{X^t_{ij}}{a^t_{ij}} \right] \quad \text{for all } j \text{ and } p_x \quad \begin{array}{l} i = 1 \ldots m+q \\ t = p \ldots p+s_j \\ j = 1 \ldots m \\ p = 1 \ldots n \end{array}$$

where:

X^p_j = the number of students admitted at level j in period p

X^t_{ij} = the amount of input i devoted to activity j, in period t

a^t_{ij} = the minimum amount of input i required to accommodate one student in activity j in year t[7]

m = the number of types of education considered in the model

n = the number of years in the planning period

q = the number of factors supplied from outside the educational system

s_j = the duration of course j in years.

Equation 4.1 states that admissions, X^p_j, cannot exceed the value of the smallest ratio of total inputs (X^t_{ij}) to the relevant input coefficient (a^t_{ij}).

6. The available school-by-school data on teacher student ratios and other input coefficients for Northern Nigeria exhibit a remarkably small dispersion around the mean value.

7. Many of the a^t_{ij} coefficients are zero.

The a^t_{ij} coefficients referring to inputs produced by the education system itself represent teacher student ratios for each of the types of teachers used in the model and student input ratios. The latter refer to the minimum number of leavers from level i required to admit one student to level j in time t. If level i is the "feeder" for level j then the relevant input coefficient is one.[8] The a^t_{ij} coefficients for inputs supplied from outside the system represent the marginal per student resource requirements.

Outputs appear in the system of constraint equations as negative inputs, and are computed on the basis of the total original student input multiplied by the fraction of the original students who can be expected to fall into each output category, namely, dropouts, failures, and successful leavers.

The matrix of a^t_{ij}'s, along with the output coefficients, is an intertemporal input-output system representing the intra-educational flow of teachers and continuing students along with the inputs of exogenous (primary) factors. It closely resembles an intertemporal input-output system for an entire economy with the major exception that the educational production processes are extremely time-consuming, some requiring as much as seven years between original input of a student and the eventual output of a graduate from that activity.

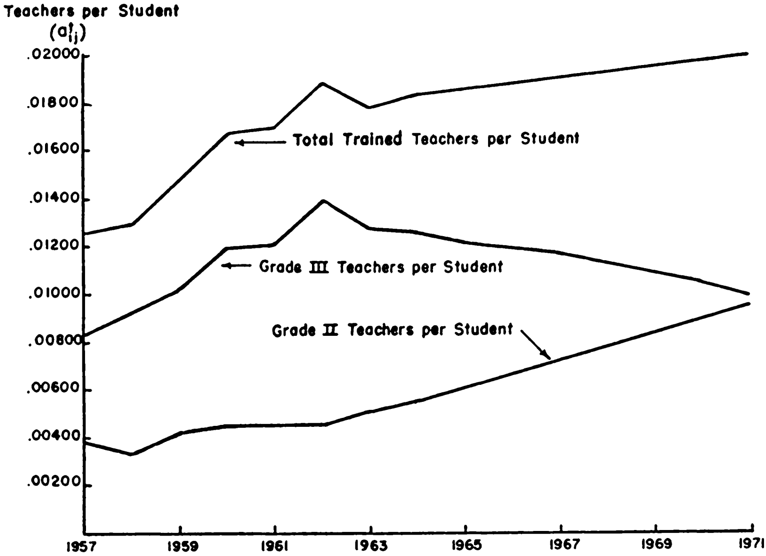
The input coefficients relating to Northern Nigeria were estimated on the basis of historical and present data on teacher student ratios (for a number of different types of teachers) and other input data. Time series of teacher student ratios were used as the basis for the projection of future changes in the teacher input coefficients. In most cases the movement of the coefficients indicates a significant improvement in the quality of the teaching staff, namely, a substitution over time of relatively well trained for less well trained teachers. An illustration of this process of technological change can be seen in Figure I, which presents the estimated values of the primary school teacher input coefficients over the years 1964 to 1971.

The resource constraints relate to three types of use:

1. use of inputs generated by the educational system itself which are defined in stock terms (i.e., teachers);

2. use of the endogenously generated inputs which are defined as flows (i.e., continuing students);

8. In one case (Northern Secondary Teachers College) the student inputs are of two different types, secondary school-leavers and Grade II teachers. In this case the student input coefficients relating to these types of students have been set at fractional values representing the student input structure of this particular institution.



Notes:
Values for 1957–64 are actual.
Values for 1965–71 are projected.
Grade III and Grade II teachers have completed 3 and 5 years of post-primary education respectively.

Historical and Projected Technological Change in the Production of Primary Education

FIGURE I

3. use of inputs supplied from outside of the educational system.

Considering the two types of constraints relating to resources produced by the educational system, recall that there are three possible uses for the output of any activity: pursuit of further education in the system, employment as a teacher in the system, or employment in the labor force outside of the educational system. These three uses can be referred to as use as an intermediate good, use for capacity creation, and deliveries for final demand. The total requirements within the educational system for labor of a given type thus depends on the levels of the activities which use it as a student input, and the required capacity creation in the activities which use it as a teacher. The total availability of individuals with each qualification is given by the numbers surviving from the base period plus the amount produced within the system or recruited from outside the system.[9] The constraint equations insure that the amount

9. We have assumed that while teachers can be recruited or imported from outside the system, continuing students must be produced endogenously.

of teachers and continuing students required by a solution does not exceed the number available.

The constraints on the use of exogenously supplied resources refer to such inputs as primary school age population and total social expenditure on education, and require that the total use of each resource not exceed the exogenously specified supply.

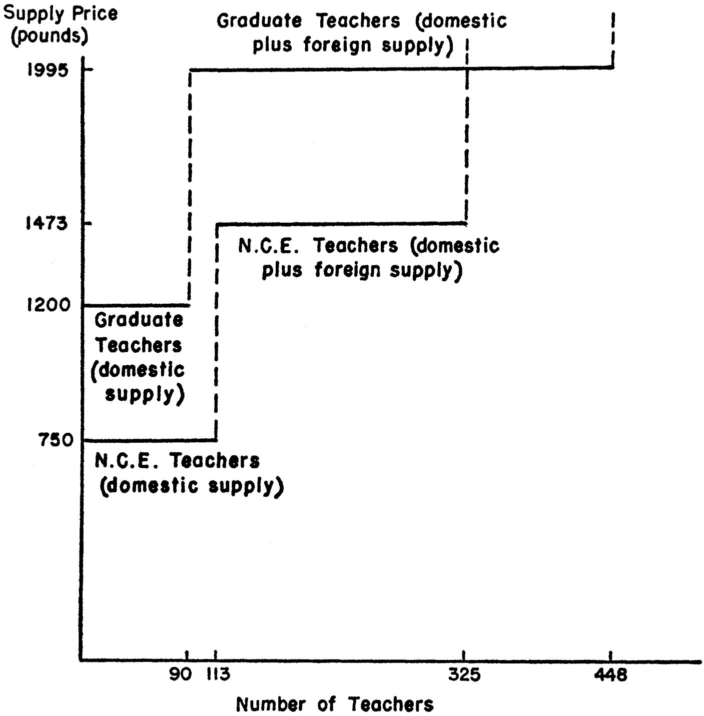
In addition to the resource constraints, boundary conditions are imposed on the instrument variables. The main considerations here were the political difficulties involved in any drastic reductions in enrollments, and the administrative obstacles to any very rapid increase.[1] The complete set of equations and a glossary of notation appear in the appendix.

Thus far we have made the usual linear programming assumption that inputs are available at constant cost up to some level beyond which they are not available at any price. An attempt has been made to modify this somewhat extreme requirement by constructing supply functions which reflect the rising supply price of the factor. The supply functions for two types of teachers, each of which may be hired locally or imported, are depicted in Figure II. The vertical distance between the first and second segment of each function is the cost of importing the teacher, namely, transport and other payments additional to the salary. The step is built into the function by allowing the system to use a new activity which imports the teacher at the indicated cost.[2] Similarly, in some runs activities allowing the recruitment of ex-teachers back into the school system have been introduced. These activities are operated at a cost based on the foregone productivity of the teacher in his non-teaching occupation; the output is the availability of additional teachers within the educational system. The introduction of these activities for the recruitment of grade II and grade III teachers (used largely in primary school) has the effect of adding a step to the present supply functions and thus reflecting the rising supply price of these inputs.

For those years immediately preceding the end of the plan period, terminal conditions must be developed so that some allowance will be made for intra-educational demand for educational outputs during the years immediately following the end of the plan period. Were this not done the system would undertake what may be called capital consumption; it would cease producing teachers and stu-

1. In the Nigerian application of the model admissions in any year were restricted to a value between 1.3 and .7 of the previous year's admissions.

2. The cost of using the teacher (salary) is charged directly to the using activity.



Notes:
a. The height of the first segment of each function reflects the cost of hiring domestic teachers (salary plus other payments). The difference between the first and second segments reflects the cost of importation of the teacher (travel costs, salary, additions, etc.)
b. The lengths of the segments are determined by the available domestic supply and the maximum limit on importation.
c. Graduate teachers refer to those holding a university degree. N.C.E. teachers and their foreign equivalents have completed 8 years of post-primary education.

Supply Functions for Graduate Teachers and Nigerian Certificate of Education (N.C.E.) Teachers in 1965

FIGURE II

dents for pursuit of further studies in the last few years of the period. A number of methods of dealing with the terminal conditions is available.[3] The method adopted here is to insure that for teacher training or the production of continuing students the activity levels immediately prior to the end of the plan period will be sufficient to

3. Two possible methods were considered but not used. First, one might have required a minimum terminal year stock of teachers of each type capable of supporting some desired (exogenously specified) rate of post terminal enrollments. A second possibility would have been to value the terminal year stock of teachers (presumably using shadow prices from previous runs in an iterative process), and then to maximize some function incorporating the present maximand and the value of the terminal year stock of teachers.

support post terminal rates of growth similar to those established during the planning period.[4]

V. The Pattern of Enrollments and Resource Use Within the Educational Sector

Only a small portion of the body of results generated with the model will be discussed below;[5] emphasis will be directed to the types of insights into concrete policy problems which can be gained with the aid of this approach to educational planning.

Before considering the actual solutions, it should be pointed out that the production side of the model alone is sufficient to generate alternative patterns of enrollments which are both internally consistent and which do not violate the exogenously specified resource constraints. Moreover, the inverse of the matrix of input and output coefficients is a convenient summary of the available educational technologies, and allows the computation of the direct and indirect input requirements for a unit of final delivery of each type of labor to the labor force. Thus we can solve a number of planning problems without reference to the discounted future earnings stream attributable to education. The objective function provides one (but not by any means the only) method of selecting a desirable solution from the multitude of feasible solutions.

This section will present some of the results concerning enrollments in the various types of schools. The following three sections will deal with the choice of techniques, the optimal total resource use by the educational system, and the pattern of importation of educated labor.

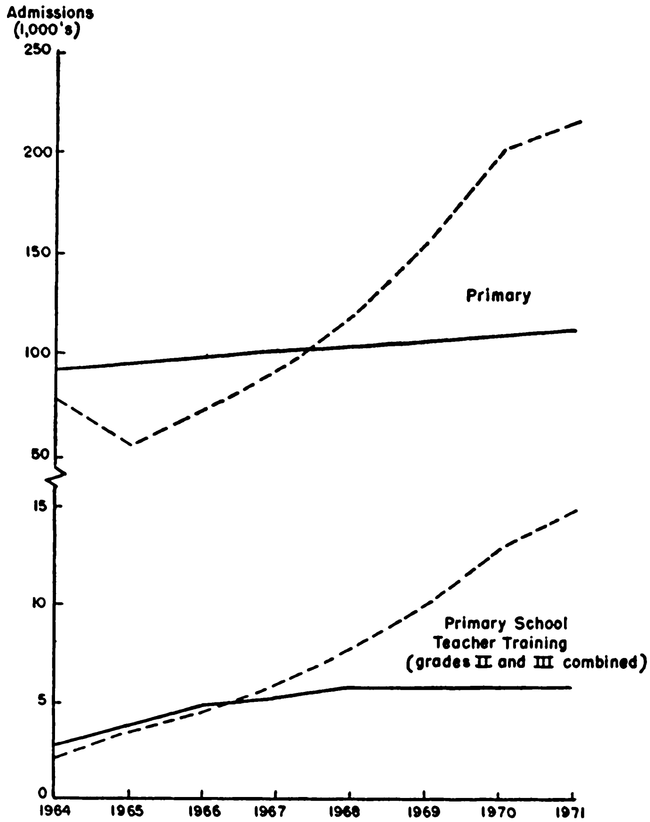
Solutions to the model yield values for each of the instrument variables relating to the admission of students to each type of school in each year of the planning period. We shall confine our attention here to primary education and related activities.

The present Northern Nigerian educational plans call for a very gradual increase in primary school enrollments accompanied by

4. The choice of terminal conditions is somewhat arbitrary. It should be pointed out, however, that while the values of the instrument variables for the last few years in the planning period may be sensitive to the choice, the relevant years are those on which immediate action must be taken. The sequential application of the model suggested in Section II obviates the need for taking action on any but the first, or the first and second, year of the plan period.

5. Well over 100 solutions of the model have been computed using alternative assumptions concerning policy, technology, and the future demand for educated labor. A more complete description of the results will appear elsewhere.

gradual increases in the associated teacher training institutions, as indicated in Figure III.[6] The model, using much of the same data, yields a radically different pattern of growth, shown also in Figure III.[7] The rapid rate of growth of primary education over the entire



Notes:
Dotted lines indicate admissions levels specified by the model.
Solid lines indicate admissions levels in current Northern Nigerian plans.

Primary School and Teacher Training Admissions 1964–1971

FIGURE III

6. Given the planned upgrading of the primary school teaching staffs, the admissions levels in current government plans (Figure III) are inconsistent. The demands for grade II and grade III teachers derived from the planned primary school admission in the early years of the plan appear to be considerably in excess of current availabilities plus planned outputs. Only a major program of recruitment of ex-teachers could render the existing plans feasible.

7. The planned admissions figures represent the outcome of a comprehensive planning process which took into account a number of noneconomic aspects of the problem not considered in this model. Thus the figures are not strictly comparable.

eight year period reflects the high ratio of net benefits to both social cost per student and inputs of teachers in the primary school activity. More explicitly one can say that the strong claim on resources exerted by primary education is due to a great extent to the low opportunity cost of its major inputs; the opportunity cost of student time is zero and the opportunity cost of grade II and grade III teachers in the economy is minute compared with the opportunity cost of university graduates, who form the bulk of teaching staffs at the post-primary institutions.

The initial decline in primary school admissions indicated in Figure III is explained largely by the required upgrading of the primary school teaching staffs and the rather complicated interrelations between primary schools and teacher training. We have found that there are a number of activities within the educational system which are particularly closely intertwined, and that the reciprocal and even multilateral trading of continuing students and teachers often results in a somewhat unexpected pattern of optimal educational growth. The connection between primary education and the two major types of primary school teacher training (grade II and grade III) is a good example of this problem.[8] Primary school-leavers are an input into grade III teacher training courses (see Table I). The outputs of the grade III course are delivered back to the primary school as teachers, or to the grade II training course for further training. Those who successfully complete the grade II course serve as teachers in the primary schools or as student inputs into the higher teacher training institutions (N.S.T.C.). Thus, while it is not exactly true that everything depends on everything else (this particular whirlpool of interdependence appears to be relatively self-contained), each activity level depends on a number of others, often in a rather complicated way.

Recall that as part of the program of quality improvement in primary school teaching the relatively well trained grade II teachers are being substituted for untrained and grade III teachers. The upward movement of the grade II teacher input coefficient over time requires that in addition to training teachers to accommodate the increment in total enrollments, the grade II teacher course must train a sufficient number to effect an increase in the grade II teacher coefficient, not only for the increment in enrollments but for the entire stock of primary students currently in the process of being educated.

8. Grade III teachers have three years of post-primary education and are the lowest category of trained teachers in the primary schools. Grade II teachers have a total of five years of post-primary training.

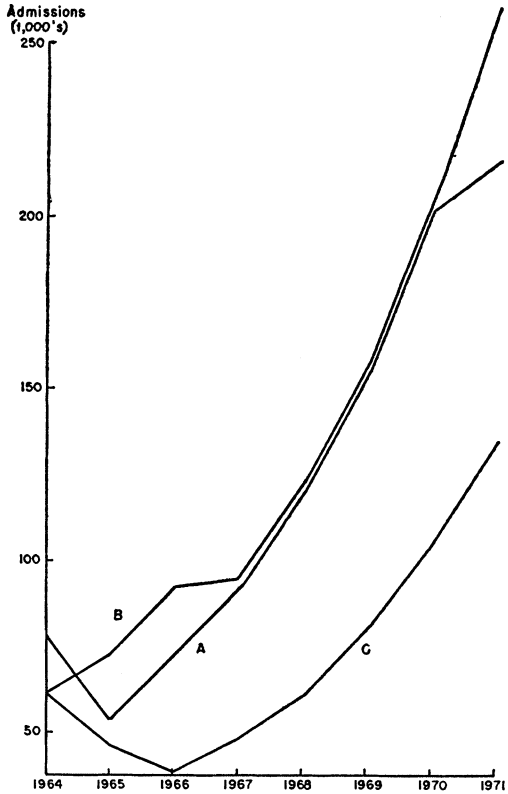
The educational system is given the choice of four alternative methods of acquiring the necessary grade II teachers:

1. admit primary school-leavers into the (three year) grade III course and admit those who successfully complete the course to the grade II course;

2. withdraw grade III teachers from teaching in primary school and admit them to the grade II course;

3. recruit ex-grade III teachers from the nonteaching labor force, and admit them to the grade II course; and

4. recruit ex-grade II teachers from the nonteaching labor force.



Notes:
A refers to the basic run presented in Figure III.
B refers to a run in which there is no upgrading in the qualifications of the primary school staff.
C refers to a run in which the recruitment of ex-teachers from the nonteaching labor force was not allowed.

Primary School Admissions with Various Policy Assumptions

FIGURE IV

All four methods are used. However, it is the withdrawal of grade III teachers from primary school for further training which is largely responsible for the early fall in primary school admissions. The process is analogous to a temporary cutback of production to allow retooling of the existing capital stock, followed by a rapid expansion with a new technology. Were the system restricted to channeling school-leavers through the usual grade III and grade II sequence, a total of five years would elapse before an increased volume of grade II output could be made available to the primary schools. In this case either admissions would have to be significantly reduced, or the upgrading of the primary school teaching staff would have to be postponed, or both. A number of runs in which recruiting ex-teachers from the nonteaching labor force was not allowed resulted in a much more pronounced and more prolonged reduction in primary school admissions. On the other hand, a run incorporating no temporal change in the teacher/student ratio (no upgrading of the primary school staffs) resulted in a monotonically increasing admissions level for primary education (see Figure IV).

VI. The Choice of Educational Techniques

Many of the policy decisions facing planners in the field of education concern changes in educational technologies. In this section we shall explore the economic implications of a number of technological changes in primary education.

The Ministry of Education in Northern Nigeria has recently given consideration to a proposal which would reduce the number of years in the primary school course. The proposal for a shorter course offers the same number of classroom hours as are presently offered over the seven year course. This is possible because of the relatively short school year in the present system. The optimality of a similar proposal has been considered with the model. Primary school activities of five years duration have been introduced. The *annual* costs are somewhat higher (to allow for the opportunity cost of withdrawing the teaching staff from possible vacation time employment) but given the reduction of the course from seven to five years, the total discounted cost is not increased. The teacher/student ratios are unchanged, except that the elimination of the sixth and seventh year obviously releases a significant portion of the teaching staff. Once the system is in operation, overall teacher requirements are reduced to five-sevenths of the previous level.[9] In addition, the

9. If one took account of the effect of wastage on the teacher/student

availability of the primary school output two years earlier increases the present value of the benefits stream.

The effect of the introduction of the new primary school course can now be outlined. The optimal primary school admissions levels are significantly increased. Moreover, the net benefits generated by the optimal solution are more than 10 per cent higher at the present level of expenditure on education (see Figure VI). Despite the increase in primary school admissions, the teacher training activities are run at virtually the same levels as in the solution with the seven year primary school course. Both the increase in total net benefits and the increase in optimal primary school admissions can be explained by:

1. the reduction in overall teacher requirements which, among other things, facilitates the "retooling" process; and
2. the increase in the present value of net benefits per student.

A number of other runs have tested the implications of the following types of structural or technological change in the production of education; all resulted in significant increases in the value of the objective function:

1. an increase in the university course from three to four years, accompanied by the elimination of the present Sixth Form, the two year university preparatory course;
2. changes in the failure rates in various teacher training activities;
3. a less rapid quality improvement in the teaching staffs in primary schools;
4. various changes in the productive techniques at the primary school level.

A particularly interesting experiment under the last heading was to allow the model to substitute equipment (texts and audio-visual materials) for the lowest grade of teachers in primary school (grade III), and to allow some substitutability between different types of teachers in the production of primary education. Using constant marginal rates of substitution between grade II and grade III teachers and between grade III teachers and equipment, over a limited range, an optimum pattern of enrollments and substitution was generated. Some factor substitution was optimal in all years of the planning period.

ratios, the reduction in overall requirements would be somewhat less. It should be pointed out that the impact of the change is not felt in the model until the sixth year of the plan, because it is assumed that primary school students already in school at the beginning of the plan will remain for the usual seven years.

VII. Optimal Total Resource Use by Education

We turn now to the question of the total resource use by the educational system. We have two related types of measures of the optimality of the division of resources between education and the rest of the economy:

1. the amount of additional resources recruited into the educational system in the optimal solution and
2. the shadow prices of resources.

The activities which recruit new factors (e.g., recruiting ex-teachers back into the educational system) will be run at positive levels whenever the indirect effect of an additional unit of resource on the discounted value of future GNP is greater than the estimate of the resource's unit cost.[1]

In all solutions of the model it has been optimal to augment the existing factor supplies with recruits both from the Nigerian labor force outside of education, and from abroad. Thus, for example, the high level of recruitment of grade III teachers reflects the fact that the marginal productivity (in terms of discounted future GNP) of grade III teachers when used in the production of primary education is considerably higher than the direct productivity of these personnel when employed in the rest of the economy. The high levels of importation of foreign teachers indicate that during most years of the planning period the value of the marginal product of these teachers within the Nigerian educational system exceeded the rather substantial importation costs.

The shadow prices of each resource provide some indication of the optimal total resource use by the educational system. If the shadow price of the resource within the educational system, measuring the direct and indirect contribution of a unit of the resource to discounted future GNP exceeds the marginal productivity of the resource in its next best use, then we can conclude that the allocation of more of the resource in question to the educational system would increase the present value of future GNP.

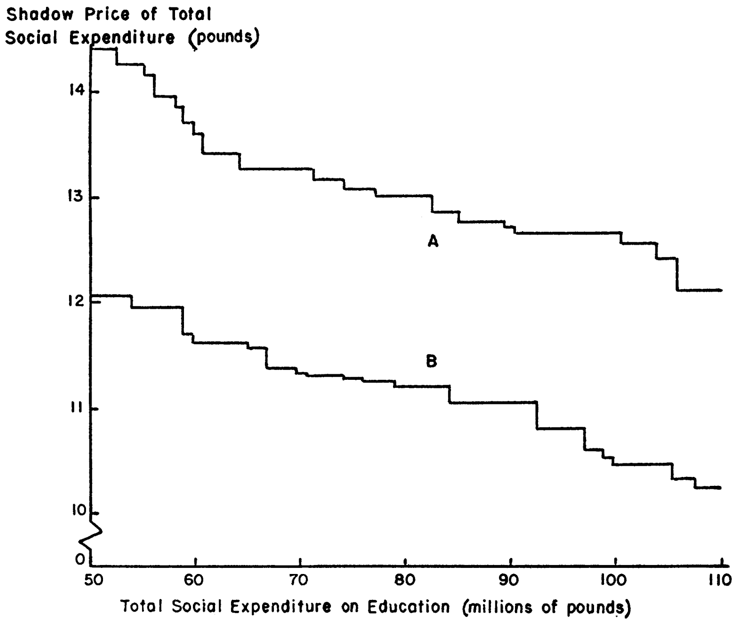
1. The unit cost of imported teachers is the additional salary and other associated costs; the unit cost of additional factors recruited from other sectors in the economy is the factor's marginal productivity in its alternative use.
Where z is a row vector of the objective function coefficients, B the basis of included activities, a_k the vector representing the recruiting activity, and c_k the estimated opportunity cost of recruitment, the simplex criterion insures that the recruiting activity will be run at positive levels whenever:

$$c_k \leq zB^{-1}a_k$$

The term on the right-hand side of the inequality is the direct and indirect effect of the availability of an additional unit of the resource on the objective function.

Total resource use in the model is measured in money terms and referred to as total social expenditure on education. This quantity includes the direct social costs of education along with the opportunity costs of students' time incurred during the process of education. In all solutions of the model the shadow price referring to total social expenditure on education is high relative to any plausible estimate of the marginal productivity of resources in alternative uses. At first glance one would conclude that a major increase in the availabilities of resources for the educational system is called for. However, the skeptic and the planner may wish to investigate how the shadow price is affected by changes in the availability of resources to the system.

Parametric programming has been used to estimate the marginal productivity function for expenditures on education. The element in the constraint vector referring to the maximum total expenditure

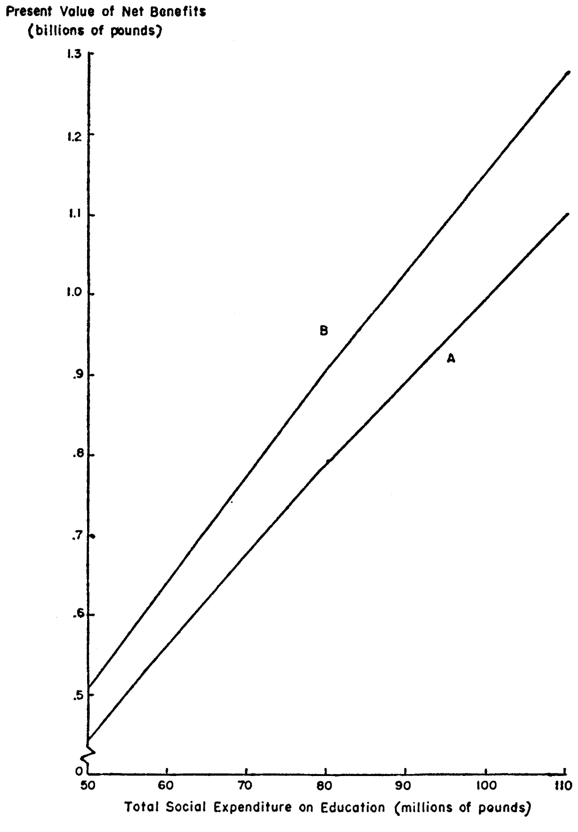


Notes:
Present value of total social expenditure is based on a 5 per cent discount rate.
The function indicated by A refers to the run using a shortened (5 year) primary school course.
The function indicated by B refers to the run using the existing (7 year) primary school course.
Current planned expenditure is in the neighborhood of £80 million.

Shadow Prices as a Function of Total
Social Expenditure on Education

FIGURE V

has been first set at a low level and then increased. At the point where each change in the optimal basis occurs, an entire new optimal solution, including the total benefits, the shadow prices and the optimal activity levels has been recorded. This technique allows us to trace out both the marginal productivity function for expenditure on education, and a function relating the total benefits to total expenditure. The two functions appear in Figures V and VI. The shadow prices appearing in the step functions in Figure V are clearly the slopes of the minute line segments which make up



Notes:
Present value of net benefits and total social expenditures are based on a 5 per cent discount rate.
A refers to the present system with a 7 year primary school course.
B refers to the revised system with a 5 year primary school course.

Present Value of Net Benefits as a Function
of Total Social Expenditure on Education

FIGURE VI

the total benefits function in Figure VI.[2] The range of variation of the total social expenditure on education presented here is centered on £ 80 million, which is about what present government plans imply. Variations beyond the range presented in the tables were thought to be of dubious value because the linearity of the relationships in the model is open to serious question when very major changes in allocation are being considered.

Two aspects of Figures V and VI are particularly striking: the high level of the shadow prices over a wide range of expenditure on education, and the very favorable ratio of net benefits to total costs. These results seem to confirm the earlier impression that a revision of the present division of resources between education and the rest of the economy in favor of education would significantly increase the present value of future GNP.

VIII. The Importation of Educated Labor

The number of foreigners involved in teaching a nation's youth is naturally a question of political as well as economic importance. The replacement of foreign by indigenous teachers is a major policy goal in a number of countries; others have explicit or implicit limits on the proportion of teaching positions which may be held by aliens. Yet foreigners are often a crucial element in expanding the supply of teachers, particularly as a temporary measure to break bottlenecks in teacher training itself. The optimal importation of foreign teachers thus depends on a trade-off between income (and perhaps other) gains made possible through a more rapid expansion in educational facilities and welfare losses occasioned by an increased dependence on foreigners.

We may expect the social welfare function to contain a negative term relating to the number of imported teachers in the school system. We may write:

$$(8.1) \qquad W = W(Y, F, \ldots) \qquad \begin{array}{l} \partial W/\partial Y > 0 \\ \partial W/\partial F < 0 \end{array}$$

where W = the social welfare function
F = total number of teachers imported
Y = the present value of future national income.

In view of the fact that over some ranges of importation foreign

2. Although difficult to detect visually the functions in Figure VI are concave from below; the implied diminishing marginal productivity is clearly shown in the negative inclination of the step functions in Figure V.

teachers contribute to the expansion of educational output and hence of future national income, we can further write:

(8.2) $Y = g(F)$ (all other inputs constant)

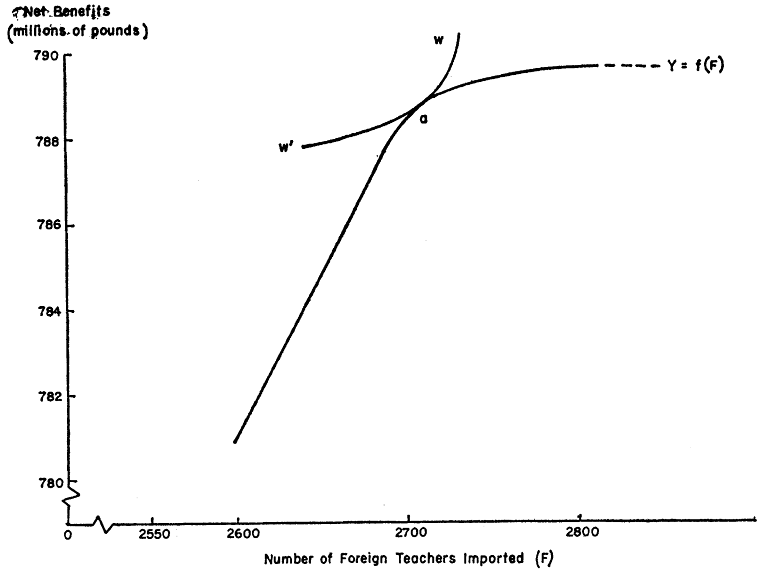
and therefore,

(8.3) $W = W[g(F), F]$.

First order conditions for the maximum W require that

(8.4) $$\frac{-\partial W/\partial F}{\partial W/\partial Y} = \frac{\partial Y}{\partial F}$$

or that the negative of the marginal rate of substitution in the social welfare function between income and foreigners must equal the marginal product of foreigners or the marginal rate of transformation of foreigners into income.[3]



Notes:
ww' represents the (hypothetical) social welfare function.
For $F > 2812$ the shadow price of foreign teachers is zero.
For $F < 2597$ no feasible solution exists.

Net Benefits as a Function of the Number of Foreign Teachers Imported

FIGURE VII

3. For simplicity of presentation, we have here ignored the term $\frac{\partial W}{\partial E} \cdot \frac{\partial E}{\partial F}$ which would take account of the fact that increased importation of foreigners allows an expansion of enrollment (E) which may be valued directly in the social welfare function, apart from the associated income gains.

We can estimate the relation (8.2) using the parametric programming technique described in the previous section. The term $\partial Y/\partial F$ is the shadow price of foreign teachers in the model, or the slope of the function appearing in Figure VII. The shape of the function and the limited range of variation of F between the point of redundancy and the point at which no feasible solution exists suggests that, given the present structure of the system, the productivity of foreigners is high at present levels of use, but that any major increase in importation would quickly depress their marginal product to zero. Nonetheless, the high shadow price of foreigners over the relevant range is suggestive of a rather major opportunity cost of pursuing nationalistic educational policies. The dotted line WW' in that figure represents a hypothetical social welfare function which yields an optimum as described in (8.4) at point a.

IX. Conclusions

Because the model employs linear constraints and a linear maximand, there are relatively few computational problems involved in solving and using the model. However, computational simplicity has been gained at the cost of a number of assumptions which do not strictly correspond to the reality of any concrete planning situation. The following limitations of the model arising from the use of these simplifying assumptions are particularly important.

First, the maximand is a linear function of the activity levels; thus the net benefits coefficients must not be a function of the level of output of any of the activities. Strictly speaking, this requires that the elasticity of demand for labor is infinite and the cross derivatives (with respect to the various labor inputs) of the production functions in the economy are zero.[4] This assumption is at the opposite extreme from that made or implied by the manpower requirements school of educational planning, namely, that the price elasticity of demand for labor is zero. The problems mentioned here are attributable to the fact that we are dealing with a sectoral model rather than with a model of the entire economy. Ideally we would use a model of the educational system and the economy in which the demand for educated labor and the supply of inputs to education are generated endogenously.

4. In the absence of strict conformity with the requirement, approximate constancy of the present value of the outputs may result from the interaction of a number of influences, for example, the expansion of the supply of educated labor accompanied by a rightwards movement of the demand curve for educated labor as a result of economic growth or technological change.

Second, in the empirical implementation of the model it has been necessary to use estimated future earnings streams as the basis for the objective function. This approach relies on the assumption that workers are paid according to their marginal productivity. In addition, the use of observed earnings as a basis for the estimation of future earnings streams rests on the assumption that the real absolute differences in the earnings accruing to labor educated to different levels and with a given number of years of experience will remain constant over time.[5]

Third, it is assumed that the observed income differentials can be attributed entirely to differences in education. This is clearly not the case if intellectual and physical aptitudes, parental wealth, or various socio-psychological attributes which are positively correlated with an individual's future earnings are also positively correlated with the likelihood of his getting an education.

Fourth, even if the first three assumptions were close approximations of reality, it should be pointed out that the observed earnings measure the private marginal productivity to the individual or to the firm rather than his social marginal productivity. The external effects of an individual's education have been omitted.

Fifth, the objective function measures only those effects which result in higher earnings. The benefits which have been defined above as noneconomic, namely, those which affect the nonincome terms in the social welfare function, are not included in the objective function.

The usefulness of a linear model of the type proposed here depends on how closely the assumptions and structure of the model approximate reality in any given planning situation and on how sensitive the results of the model are to a likely degree of error. On the basis of sensitivity analysis of the model with respect to each of the above assumptions it can be said that the results for Northern Nigeria are not significantly affected by plausible alternative assumptions. The same general conclusion applies to reasonable changes in the data underlying the parameters of the model. Similarly favorable results were yielded by sensitivity analysis of the choice of a time discount rate and the estimated rate of unemployment among the outputs of the educational system.[6] A run in which

5. It should be pointed out that in the presence of general increases in output per worker, constancy of the absolute differences in earnings is consistent with a narrowing of relative earnings.

6. The sensitivity tests and a more complete discussion of the empirical importance of the limitations of the model are found in Samuel Bowles, "The Efficient Allocation of Resources in Education," *op. cit.*, Appendix 6.4 and

only 60 per cent of the earnings differentials by educational level were attributed to education produced no major qualitative changes, although net benefits were naturally reduced.

Despite the very real nature of the above shortcomings of the model, this approach to the economics of educational planning does yield a wealth of insights into the question of optimal resource allocation in education. By making explicit the complicated interrelations within the educational system it allows the investigation of the direct and indirect effects of a multiplicity of concrete policy choices. The model facilitates the consideration of the efficiency of alternative educational production processes simultaneously with the choice of levels of production. The shadow prices generated by the model are useful in identifying major resource scarcities and in suggesting the relative importance of policy measures to alter educational technologies or the structure of the educational system. Lastly, the model has been constructed so as to rely on data which are either available in most countries or can be easily generated.

Appendix and Glossary of Notation

Outline of the Structure of the Model as Applied to Northern Nigeria

I. The Planning Period: Eight years extending from 1964 through 1971.

II. Activities: In most runs, a total of 120 activities, or one per year for the following:

A. Activities making deliveries to the labor force:
1. primary school
2. secondary school
3. technical training school
4. form VI (college preparatory)
5. university education in Nigeria
6. university education abroad.

B. Activities devoted exclusively to teaching training or to the preparation of students for further courses:
1. craft school (preparation for technical training school)
2. grade III teacher training
3. grade II teacher training
4. Nigerian Certificate of Education teacher training.

Appendix 7.1. Some of the insensitivity to plausible parametric variations may be explained by the upper and lower bounds on activity levels.

C. Activities importing and recruiting teachers:
1. importing foreign teachers holding university degrees
2. importing foreign teachers holding the equivalent of a Nigerian Certificate of Education
3. recruiting ex-grade II teachers from the labor force
4. recruiting ex-grade III teachers from the labor force
5. recruiting additional senior university teachers from abroad.

III. THE OBJECTIVE FUNCTION: The terms in the objective function measure the net contribution of each activity to the present value of future national income, as defined in Section III.

Using the notation as defined in the glossary, the maximand is:

$$Z^* = \sum_{j=1}^{m} \sum_{p=1}^{n} X^p{}_j(Y^p{}_j - Y^p{}_{j'} - C^p{}_j).$$

IV. CONSTRAINTS:

A. Constraints on the use of inputs which are defined as stock and which are generated within the educational system, namely, teachers:

$$\sum_{j=1}^{m} \quad \sum_{p=t+1-s_j}^{t} a^t{}_{ij} X^p{}_j - \sum_{p=1}^{t=s_j} g_i X^p{}_i - X^t_{i*} \leq B^t{}_i$$

The first term of the expression is the total enrollments in activity X_j at time t, multiplied by the required input of teachers of type i per student in activity j, summed over all of the m activities. The second term is the total output since the beginning of the planning period of the teacher training activity producing resource i (adjusted for failures and dropouts). The third term is the total importation or recruitment of teachers of type i from outside the educational system in time t. The right-hand side term is the total stock of the type i teachers in the system in the first year of the planning period who have remained in the system (i.e., who have not retired) up to year t. Thus the above set of equations requires that total use of type i teachers not exceed the available supply for each type of teacher in each year of the planning period.

These constraints are thirty-two in number corresponding to one per year for the following inputs:
1. grade III teachers
2. grade II teachers
3. Nigerian Certificate of Education teachers
4. university graduate teachers.

B. Constraints on the use of inputs which are defined in flow terms and which are generated within the educational system, namely, students.

$$\sum_{j=1}^{m} a_{ij}X^t{}_j - g_iX_i{}^{t-s_i} \leqq 0.$$

The first term of this constraint is the total students with qualifications i required as inputs into educational processes in time t, while the second term is the total output of the activity producing these students at the end of the previous year. This set of equations thus requires that the intake of students into a given type of school in time t must not exceed the previous year's output of students with the prerequisite qualifications for entry.

These constraints are thirty-two in number corresponding to one per year for the following inputs:

1. primary school-leavers
2. craft school-leavers
3. secondary school-leavers
4. form VI leavers.

C. Constraints on the use of exogenously supplied inputs:

$$\sum_{j=1}^{m} \sum_{p=t+1-s_j}^{t} a^t{}_{ij} X^p{}_j \leqq B^t{}_i.$$

The first term is the total enrollments in time t in type j schools, multiplied by the per student input requirement, summed over the m types of education. The right-hand side term is the exogenously specified total availability of resource i in time t.

These constraints are seventeen in number and refer to the following inputs:

1. present value of total social expenditure on education (only one constraint for all eight years)
2. senior university teachers
3. children in the six year age group.

D. Boundary conditions for admissions levels:

$X^p{}_j \geqq .7X^{p-1}{}_j$

$X^p{}_j \leqq 1.3X^{p-1}{}_j$

for recruiting and importing activities:

$X^p{}_{i*} \leqq R^p{}_{i*}$

V. A Glossary of Notation

Notation relating to the instrument variables:

$X^p{}_j$ = the number of students admitted to level j in period p: $j = 1 \ldots m, p = 1 \ldots n$.

m = the number of activities.

n = the number of years in the planning period.

X^t_{i*} = the imports of resource of type i in period t.

Notation relating primarily to the constraint equations:

$a^t{}_{ij}$ = the minimum input of resource i in period t required to accommodate one student in activity j: $t = 1 \ldots n, j = 1 \ldots m, i = 1 \ldots m + q$.

q = number of exogenously supplied inputs.

B^t_i = the amount of resource i available to the system in time t.

s_j = the length of course j in years (similarly defined for s_i).

X^t_{ij} = the amount of input i devoted to activity j in period t.

R^p_{i*} = upper limit on the recruitment or importation of teachers with qualification i in period p.

Notation relating primarily to the objective function:

Z^p_j = the net benefits function coefficient associated with activity X^p_j.

z = the row vector $(1 \times nm)$ of net benefits coefficients Z^p_j.

Y^p_j = the present value (discounted to year 1) of the earnings accruing to an output of activity X^p_j.

$Y^p_{j'}$ = the present value (discounted to year 1) of the alternative earnings stream; namely, that which would have accrued to the individual had he not received education at activity j.

C^p_j = the present value (discounted to year 1) of the per student cost of operating activity X^p_j for the entire course of s_j *years*.

g_j = the fraction of the total admissions to activity X_j which is expected to complete successfully the course.

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